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Sovereign Debt Overhang and Monetary Policy*

Frank Smets and Mathias Trabandt

Abstract:

The financial crisis and the subsequent world-wide recession has led to a ballooning of government debt: In the advanced economies public debt is now approaching 100 percent of GDP, levels that are unprecedented in peace time. High and rising government debt complicates monetary policy. First, it increases the burden on monetary policy to stabilise the economy, which may be more difficult or uncertain in an environment where conventional monetary policy is constrained by the zero lower bound on nominal interest rates. Second, it increases the pressure to inflate, thereby possibly undermining the credibility and the independence of the central bank to maintain price stability. Third, it increases the perceived riskiness of government debt and may thereby undermine the proper functioning of financial markets and the transmission process of monetary policy, as has been evident in the euro area. This paper reviews some of these issues concerning the interaction between high government debt and monetary policy.

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1. Introduction

The financial crisis that fully erupted following the failure of Lehman Brothers and the subsequent worldwide recession have triggered a rapid, large and at times coordinated response of monetary and fiscal authorities across the world. As a result, nominal short-term interest rates are close to their effective lower bound in the major industrial countries, central bank balance sheets have increased very significantly and government budget deficits and public debt have ballooned. On average public debt in the advanced economies is now reaching 100 percent of GDP, levels that are unprecedented in peace time.[1] The rise in government debt raises concerns about the sustainability of public finances and the implications for the growth outlook. For example, Reinhart and Rogoff (2010) and Cecchetti et al (2011a) have documented that historically public debt ratios of more than 80-90% typically are associated with a long subsequent period of low growth. Taking into account the large and rising fiscal costs related to an aging population, Cecchetti et al (2010) conclude that the path pursued by fiscal authorities in a number of industrial countries is unsustainable. In a number of euro area countries the rapidly growing government debt has led to rising interest rate spreads, setting in motion a self-fulfilling negative spiral whereby rising spreads increase the interest rate burden, thereby reinforcing the increase in debt and justifying a further rise in spreads, in turn creating systemic risks in the euro area as a whole. Against this background, the need for fiscal consolidation has quickly become one of the top priority policy challenges in many countries.

The rising government debt also complicates monetary policy. First, to the extent that the necessary fiscal consolidation programmes have a negative short-term impact on economic activity and constrain an active use of fiscal policy including the automatic stabilisers, it puts a larger burden on monetary policy to stabilise the economy.[2] This may not be straightforward, if standard monetary policy is constrained by the zero lower bound on nominal short-term interest rates. In that case, non-conventional measures including the large-scale purchase of government bonds may have to be

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[1] See IMF (2011).

[2] There is a debate about the extent to which and under what conditions fiscal consolidation can have non-Keynesian positive effects. See, for example, Alesina and Ardagna (2010). The consensus is, however, that in most cases one needs to go through some short-term pain to have a long-term gain. See, for example, Clinton et al (2010) and IMF (2010).

used, but their effectiveness is uncertain. Second, to the extent that long-term government debt is issued in nominal terms it increases the pressure to reduce the real value of the debt by unexpected inflation. Inflation may also reduce the real burden of some of the nominal entitlement programmes.[3] High government debt may also increase the pressure to rely on alternative sources of government finance such as seignorage. These pressures risk undermining the credibility and the independence of the central bank to maintain price stability and may thereby give rise to higher inflation expectations. Finally, the increasing riskiness of government debt may undermine the proper functioning of financial markets and the transmission process of monetary policy. For example, by reducing the value and quantity of safe collateral it may increase the price of risk and liquidity premia. Moreover, to the extent that government interest rates set a floor for the cost of financing of private firms and households in the country it increases the cost of private financing and complicates the transmission of monetary policy. Finally, a reduction in the value of government bonds will reduce the capital ratio of banks holding these government bonds and may thereby lead to a credit crunch as those banks try to adjust and deleverage.

In this paper, we review some of the issues related to the interaction between high government debt and monetary policy. In section 2 we first briefly review, by way of background, the fiscal and monetary policy response in the euro area, the United States and the United Kingdom. This section highlights that there are many similarities in the monetary and fiscal response in those three areas, but also some differences. In all areas, policy-controlled interest rates were rapidly reduced towards the effective lower bound. However, the increase in the government deficit and debt as a percentage of GDP was larger in the US and the UK than in the euro area, and in the former countries the increase in the size of the central bank balance sheet involved a larger share of purchases of government securities. In this section, we also briefly review the sovereign debt crisis in the euro area. Following the convergence of sovereign bond spreads in the first decade of EMU, spreads have widened dramatically in particular since the large upward revisions in Greek debt and deficit numbers at the end of 2009. A review of the empirical literature confirms that before 2007 sovereign bond spreads were only weakly related to fiscal fundamentals. In

[3] See, for example, Persson et al (1996).

contrast, after 2008 both sovereign bond and CDS spreads became increasingly and possibly excessively, sensitive to large changes in government debt. Evidence of contagion both among government bond markets in the euro area and between the sovereign and the banking sector, leading to a malfunctioning of the monetary policy transmission process in the euro area, led the ECB to establish its Securities Market Programme in May 2009.

Against this background, we use in Section 3 a Blanchard-Yaari overlapping generations model with sticky prices to analyse the effects of higher public debt following a large recessionary shock. Using the Blanchard-Yaari framework, Devereux (2010) emphasises that debt-financed fiscal policy may have significantly larger multiplier effects in a non-Ricardian world and when interest rates are constrained by the zero lower bound. We extend his analysis by showing that a debt-financed increase in lump-sum transfers will also be expansionary. However, the quantitative impact very much depends on whether the short-term interest rate is constrained by the lower bound and the horizon of the overlapping generations. A growing literature has documented quite significant announcement effects of LSAPs on bond yields.[4] The mechanisms through which this happens are, however, less well understood. The effects of a classical open market operation whereby the central bank issues money to buy government bonds continue to be small in our framework.[5] However, non-conventional policies such as a money-financed fiscal expansion or a commitment to keep interest rates at zero for longer can have non-negligible economic effects. There are, however, clearly limits to the extent to which debt financing can help the real economy. Using the Blanchard-Yaari model we show that a loss of confidence in the government bond market may have quite significant negative effects on economic activity and inflation, in particular when the nominal interest rate is constrained at zero. Introducing an endogenous sovereign debt premium that rises with increasing public debt, we find that the recession may be quite a bit deeper and a fiscal policy geared at limiting the rise in government debt may be beneficial to the economy.

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[4] For a recent overview see, for example, Kozicki (2011).

[5] See Eggertson and Woodford (2003) and Curdia and Woodford (2011) for a discussion of the irrelevance result of traditional open market operations at the zero lower bound.

The previous analysis is done under the assumption that monetary policy pursues an inflation objective and the fiscal authorities adjust primary balances to target a certain debt level. In other words, in Leeper's (1991) terminology the economy is operating in an active monetary – passive fiscal policy regime. In reality however, the fall-out of the financial crisis has increased the probability of a switch to an active fiscal – passive monetary policy regime as interest rates are bound at zero and rising government debt has brought public finances closer to the fiscal limit, in particular if one also takes into account unfunded pension and other liabilities. In Section 4 we discuss the inflationary risks of high government debt accumulation and central bank financing. The risk-taking capacity of the central bank is limited by the need to maintain an inflation target and or the tax capacity of the government. Buiter (2007) and Durré and Pill (2011) emphasise that the central bank's exceptional creditworthiness ultimately depends on fiscal backing. When this fiscal backing is no longer sufficient, then also the central bank's credibility will be undermined. This may happen in two ways: Either by increasing the inflation tax and allowing seignorage to be an alternative source of financing; or by undermining the credibility of stability-oriented monetary policy directly. The first case corresponds to the Sargent-Wallace framework of the so-called unpleasant monetary arithmetic. In this case, an increase in government debt, if not fully backed by future real primary surpluses, will increase concerns about monetization of public debt, which will in turn raise inflation expectations and thereby increase long-term interest rates. The second case corresponds to the Fiscal Theory of the Price Level (FTPL). In this case, an increase in government debt increases the wealth of bond holders while not reducing the wealth of others. The increase in debt thereby boosts aggregate demand and pushes up the price level. In this regime, the price level is the factor that equilibrates the nominal value of future discounted primary surplus and the nominal value of public debt. In both cases, rising inflation expectations and falling nominal bond prices would be the outcome.

In Section 4 we briefly and selectively review the theoretical literature, as well as the empirical evidence about the link between government debt and inflation. This review suggests that unsustainable government finance often is the source of episodes of very high inflation, which almost universally are associated with high money growth. However, it is more difficult to detect Granger causality from government debt to inflation in the advanced economies over the last three decades, possibly reflecting the more stable fiscal and monetary policy framework. This evidence together with the current stability of inflation expectations and high bond prices should, however, be of only limited comfort, as both theoretical and empirical evidence suggests that the regime may switch quite abruptly as the fiscal space shrinks.

We end the paper by summarising our findings and the resulting policy implications in Section 5.

2. Background

In this section we briefly describe and compare the behaviour of growth, inflation, short-term interest rates, the size of the balance sheet of the central bank, the general government deficit and debt and the long-term government bond rate in the euro area, the United States, Japan and the United Kingdom during the financial crisis and its aftermath.

{Insert Chart 1}

Chart 1 shows how, following the failure of Lehman Brothers in September 2008 and the resulting collapse of the interbank market and rise in interest rate spreads, annual GDP growth collapsed with a trough of about minus 5 percent in both the euro area and the United States, and a significantly larger drop in the United Kingdom and Japan. As a result of the world-wide fall in demand, oil and commodity prices fell from their peaks in 2008 and contributed to a quite rapid fall in consumer prices which reached negative annual rates in 2009 before bouncing back in 2010, as shown in Chart 2. One exception is the United Kingdom where annual inflation remained above 1 percent partly due to a sharp depreciation of the pound sterling.

{Insert Chart 2}

In response to the rapid fall in demand in the last quarter of 2008 and the beginning of 2009 and the risks of deflation, monetary and fiscal authorities in the major advanced economies eased policy rapidly and very significantly. On the monetary policy side,

Chart 3 plots the short-term nominal interest rates in the euro area and the United States. Policy-controlled short-term interest rates were rapidly reduced to levels close to the zero lower bound. Moreover, various non-conventional monetary policy measures, which aimed at avoiding that liquidity shortages in various financial markets (in particular in the money market) translated into an outright systemic collapse, resulted in a sharp increase in the size of the balance sheet of the central bank (Chart 4) and a gradual reduction of money market spreads.

{Insert Chart 3}

In the euro area, the enhanced credit support implemented by the ECB in the course of 2009 consisted of (i) changing the provision of liquidity from variable-rate financing to full allotment at a fixed interest rate, (ii) broadening the collateral base which financial institutions could use to obtain central bank refinancing, (iii) lengthening the maturity of the refinancing operations, (iv) providing dollar refinancing through foreign exchange swaps; and (v) supporting the covered bond market which is an important source of long-term financing for financial institutions in the euro area through the Covered Bond Purchases Programme (CBPP). In addition, as the sovereign debt crisis broke out in 2010, the Securities Market Programme (SMP) consisted of the purchase of selected government bond securities to alleviate malfunctioning in the government bond market and support the transmission of monetary policy throughout the euro area. Nevertheless, the share of purchases of government securities in the increase of the central bank’s balance sheet is significantly larger in the United States and the United Kingdom due to the various LSAP (Large-Scale Asset Purchases) and QE (Quantitative Easing) programmes in those countries. As the sovereign debt crisis in the euro area intensified in 2011, the expanded liquidity provision by the ECB including 3-year Long-Term Refinancing Operations and a re-activation of the SMP led to an additional expansion of the ECB’s balance sheet.

{Insert Chart 4}

On the fiscal policy side, the deterioration of the economic outlook, discretionary fiscal stimulus programmes and to a lesser extent support to the financial sector resulted in a sharp increase in the general government deficit and a rapid rise in public debt in all four countries. Chart 5 shows that both the total and the structural government deficit increased by less in the euro area than in the United States, Japan and the United Kingdom. As a result, government debt rose more rapidly in the latter countries and surpassed the net debt to GDP ratio in the euro area in 2011. Nevertheless, long-term interest rates on government bonds fell to historic lows, partly driven by the historically low short-term interest rates and the large provision of central bank liquidity. The outbreak of the sovereign debt crisis in the euro area in 2010 contributed to a rising gap between average bond yields in the euro area and those in the United States, Japan and the United Kingdom. In line with the low nominal long-term interest rates, long-term inflation expectations have been stable throughout the crisis episode.

{Insert Charts 5, 6 and 7}

Within the euro area, fiscal developments have been quite diverse. Charts 8 and 9 illustrate the cross-country variation in debt and deficits. Following the convergence of sovereign bond spreads in the first decade of EMU, spreads have widened dramatically in particular since the large upward revisions in Greek debt and deficit numbers at the end of 2009 (Chart 10). A number of empirical papers (e.g. Arghyrou and Kontinikas (2011), Ejsing, Lemke and Margaritov (2011) and De Grauwe (2011) have documented a regime change in the determinants of sovereign bond spreads before and after the financial crisis. Before 2007 sovereign bond spreads were only weakly related to fiscal fundamentals. In contrast, after 2008 both sovereign bond and CDS spreads became increasingly and possibly excessively, sensitive to large changes in government debt. Evidence of contagion both among government bond markets in the euro area (e.g. De Santis (2011) and between the sovereign and the banking sector (e.g. Corsetti et al (2011)), leading to a malfunctioning of the monetary policy transmission process in the euro area, led the ECB to establish its Securities Market Programme in May 2009. The SMP has helped avoiding that the sovereign debt crisis turned into a full-blast systemic financial crisis, but a durable solution must build upon four pillars: i) rebuilding the confidence in the sovereign by fiscal consolidation and the establishment of sufficient fiscal buffers; ii) the further integration of the banking sector in order to cut the close link between national banking sectors and government; iii) the establishment of a sufficiently large and flexible ESM to ringfence solvent governments and avoid contagion and iv) the strengthening of the surveillance of growing imbalances within the monetary union.

{Insert Charts 8, 9 and 10}

3. Government debt and monetary policy in a Blanchard-Yaari model

In order to study the interaction between government debt and monetary policy, in this section we develop a Blanchard-Yaari-type macro-economic model along the lines of Devereux (2009). In this model, every period new households are born with a fraction $1-\delta$ of total population and die with a probability of $1-\delta$. Because households have no bequest motive, the overlapping generation nature of the population structure implies that government bonds and money are net wealth: The usual Ricardian equivalence in dynamic models with infinitely-lived households breaks down. A debt-financed increase in lump-sum transfers to households will have a positive effect on spending because a part of the government debt will be paid back by future generations. This makes the model particularly suitable for studying the impact of government debt on the economy.

The appendix lays out the decision problems of the households and firms, as well as the aggregate dynamics of the economy and its steady state. Each household consumes a bundle of consumption goods, enjoys the benefits from holding money, supplies labour and saves in the form of nominal government bonds or money holdings. There is no capital in the model. Money demand is assumed to be satiated at a specific level of real money balances. Intermediate firms produce the differentiated consumption goods using labour and set their prices in a monopolistic competitive market with price stickiness as in Calvo (1979). Price stickiness gives rise to a New Keynesian Phillips curve and implies that monetary policy has real effects in the short term. We assume that the monetary authorities follow a Taylor rule when the short-term nominal interest rate is positive and revert to a money supply rule at the zero lower bound. The monetary authority transfers part of its profits to the government and invests the other part in government bonds. The fiscal authority issues government bonds, raises distortionary labour taxes and adjusts lump-sum transfers to households in order to target a 60 percent government debt to GDP ratio.[6]

As is clear from the description of the steady state in the appendix, when $\delta < 1$, the ratio of government debt and real money held by the households to GDP will have a positive impact on the steady state real interest rate. However, at the zero lower bound when money balances are satiated, a pure open market operation consisting of a swap of government bonds for money will have no impact on the economy. Similarly, bond and money holdings enter the dynamic Euler equation of the households and will have real effects on the savings decisions of the households.

In order to investigate the interaction of the zero lower bound on interest rates and the accumulation of government debt, we use a calibrated version of the Blanchard-Yaari model to simulate a great-recession type of shock. The appendix contains the calibration of the model which aims at roughly mimicking the quantitative impact of the great recession. We assume that the economy is hit by a large and persistent rise in the discount factor. Such a rise in the discount factor can stand in for a tightening of credit constraints or increased precautionary saving due to a rise in uncertainty. The discount factor shock lasts for three years.

{Insert Chart 11}

Chart 11 plots the economy's response to the discount factor shock in the baseline version of the model. It leads to a fall in real GDP of about 15 percent and a more muted drop in inflation towards zero. As a result, the central bank lowers the short-term nominal interest rate to the effective lower bound, where it stays for a bit less than three years. The drop in interest rates leads to a rise in money demand which is accommodated by the central bank. The fall in output reduces labour tax revenues of

[6] We do not address the optimal level of debt. Recently, Leith et al (2011) analyse the optimal level of public debt in a Blanchard-Yaari model. Another interesting recent paper is Adam (2011) who analyses the implications of nominal government debt for the optimal response to productivity shocks. In his framework, higher government debt requires lowering the average level of public spending and exposes fiscal budgets to increased risks following technology shocks or – more generally – fluctuations in the tax base. These budget risk considerations can provide quantitatively important incentives to reduce government debt over time. The results in this paper suggest that debt optimally converges to zero over time and that the optimal speed of debt reduction tends to increase if governments cannot adjust their spending plans following fluctuations in the tax base.

the government, raises the deficit and generates a rise in government debt of about 15 percentage points of GDP. The rather large, but realistic increase in debt is the result of the fact that we assume that initially the government does not reduce transfers to counteract the rise in government debt. The positive impact of the rise in government debt on consumption under the Blanchard-Yaari OLG structure can be seen by comparing the effects of the shock in the baseline with those in an infinitely-lived agent model ($\delta = 1$). In the latter case, real GDP drops by an additional 3-4 percent.

{Insert Chart 12}

The non-Ricardian effects of government debt in this model are also shown in Chart 12, which plots the effect of a debt-financed increase in transfers to the household sector. In the infinitely-lived model, this has no impact on output, inflation or interest rates, as households understand that in the future transfers will have to be reduced to bring the debt ratio back to its target level. In the OLG framework, the rise in debt has a positive impact on output, inflation and the short-term interest rate as a part of the debt burden will be carried by future generations. In the baseline model, these effects are quite small. They become, however, more sizeable when the nominal interest rate is stuck at the lower bound (Chart 13) and/or the average life of households is reduced implying a lower effective discount factor (Chart 14). Devereux (2009) investigates the multiplier effects of government spending in a similar Blanchard-Yaari framework and finds that the multiplier becomes significantly greater than one with a discount factor of 0.965 and when the zero-lower bound is binding.

{Insert Chart 13 and 14}

In spite of the non-Ricardian effects of government debt, it turns out that if the government had maintained its fiscal policy rule in the initial three-year period and had systematically lowered transfers in order to limit the build up of debt, the recession would have been only mildly deeper. Of course, this result relies on the availability of lump-sum transfers. A deeper recession would have incurred if the government had increased distortionary labour taxes in response to the rise in government debt.

In a number of euro area countries, the rise in government debt has led to a rise in sovereign spreads (see Section 2). The benefits of letting the automatic stabilisers work have to be traded off against the costs of a rise in interest rate spreads due to, for example, increased probability of default. In Chart 15, we explore the effects of an expected increase in sovereign spreads, which lead to a rise in long-term government interest rates between 1 and 2 percentage points. This shock leads to a deep and persistent recession and a deflation lasting for about one year. In response the central bank lowers the policy rate to the lower bound and keeps it there for three years.

{Insert Chart 15}

Corsetti et al (2011) have investigated the effects of consolidation when sovereign spreads respond to the level of debt and have an effect on the private cost of financing. They find that such spread effects increase the probability of multiple equilibria, whereby an expected increase in spreads has a dampening effect on economic activity through a rise in the cost of financing, which in turn leads to a rise in deficits and debt justifying the initial rise. This is reminiscent of the dynamics of rising sovereign spreads, increasing costs of private financing, lower growth and weaker public finance that can be observed in some euro area countries with high and rising government debt. They also find that depending on the level of debt the multiplier of a reduction in fiscal spending may be negative, as the consolidation reduces sovereign spreads and stimulates spending.

In order to illustrate the counterbalancing effects of government debt through the sovereign debt premium, we implement an ad-hoc endogenous sovereign debt premium in the Blanchard-Yaari model. Following the empirical work of Corsetti et al (2011), we assume that the required rate of return on government debt increases in a quadratic fashion as the debt to GDP ratio increases above 60 percent[7]. Chart 16 shows the results following a rise in the discount factor. The rise in government debt following the drop in activity now has the additional effect of increasing the sovereign risk premium. This increases the return on government debt and leads to a further fall in output and inflation. As a result, the central bank keeps its interest rate at the lower

[7] See the appendix for the exact calibration.

bound for longer, but because of the lower bound monetary policy can not offset the rise in interest rates completely. In such circumstances, a fiscal consolidation policy targeted at limiting the increase in government debt may have beneficial effects.

{Insert Chart 16}

Non-conventional monetary policies may be an alternative policy instrument to reduce the output costs of a rise in government debt. This is illustrated in Charts 17 and 18. Chart 17 takes the scenario with an endogenous risk premium plotted in Chart 16 as baseline and shows the effects of a credible central bank commitment to keep policy rates at zero for three quarters longer than implied by the Taylor rule. As a result, long-term interest rates fall by considerably more and output and inflation by significantly less. These beneficial effects on the economy imply a much less pronounced increase in government debt and a much smaller rise in the risk premium in turn contributing to a smaller recession. Of course, these quite powerful results depend on the credibility of the commitment to keep interest rates low for longer.

{Insert Chart 17 and 18}

An alternative monetary policy is to boost the money supply when interest rates hit the lower bound. This is shown in Chart 18. In this case, the central bank increases the money supply by a very considerable amount of 40 percent of GDP. Again this leads to a drop in long-term interest rates, a smaller recession, less disinflation and a much smaller rise in government debt. In contrast to the previous policy experiment, in this case, the central bank will raise the policy rate from the zero lower bound earlier than in the baseline. Overall, both policies appear to work quite effectively in reducing the impact of the initial shock, even if conventional policy is constrained at the lower bound. The effectiveness is, however, based on a credible commitment for the central bank to maintain price stability and for the fiscal authorities to ultimately reduce government debt. High government debt may put those commitments in doubt. That possibility is what we turn to next.

4. Theory and evidence on the relationship between government debt and inflation

4.1. Unpleasant arithmetic and the Fiscal Theory of the Price Level

The analysis in the previous section assumed that the monetary/fiscal policy regime was characterised by an active monetary policy focused on maintaining price stability and a passive fiscal policy which adjusts the primary surplus in order to back up the value of debt. The notion that central bank independence and credibility to achieve price stability depends on a credible debt-stability-oriented fiscal policy has long been recognised. In the absence of such a stability-oriented fiscal policy, fiscal outcomes may determine inflation outcomes. This is sometimes called fiscal inflation as in Leeper and Walker (2011).

There are two basic approaches that may explain the link between unsustainable fiscal policy and inflation. The traditional and most well-known argument relies on the Sargent-Wallace framework of the so-called unpleasant monetary arithmetic whereby an increase in government debt, if not fully backed by future real primary surplus, will increase concerns about monetization of public debt, which will in turn raise inflation expectations and thereby increase long-term interest rates. This will in turn reduce money demand and push up the price level even without a contemporaneous increase in money supply. In this case, seignorage is used as an alternative source of finance.[8][9] However, seignorage is a relatively limited source of government revenue and is also subject to a Laffer curve which determines the maximum amount of revenues that the government can collect. As inflation rises, the demand for money (the tax base) will fall reducing overall seignorage income.[10]

An alternative approach is the so-called Fiscal Theory of the Price Level (FTPL). In this regime, the price level is the factor that equilibrates the nominal value of future discounted primary surplus and the nominal value of public debt. Leeper and Walker (2011) recently summarised the research on the FTPL and clarified perceptions and misperceptions of fiscal inflation. Using a simple infinitely-lived representative

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[8] Of course, as long as the transfer of seignorage is compatible with the inflation objective, as for example is the case in the model of Section 3, this should not necessarily lead to inflation expectations.

[9] This will be independent of the effects of devaluing the nominal debt and will also be a source of revenue even if the public debt is in real terms.

[10] See Buiter (2007) for an extensive discussion on seignorage and the interaction between the central bank's and governments budget constraints.

household model with a constant endowment and a government that issues nominal debt and raises lump-sum taxes to finance transfers, they show that the household's intertemporal Euler equation, the government's budget constraint and the central bank's reaction function can be combined to yield the following equations:

(1) $$\frac{\beta}{\alpha} E_t \left[\frac{P_t}{P_{t+1}} - \frac{1}{\bar{\pi}} \right] = \frac{P_{t-1}}{P_t} - \frac{1}{\bar{\pi}}$$

where P_t is the price level, $\bar{\pi}$ the inflation objective, β is the discount factor, and α the central bank's reaction coefficient to inflation; and

(2) $$E_{t-1}\left[\frac{B_t}{P_t} - \bar{b} \right] = E_{t-1}\left(z_t - \bar{z} \right) + (\beta^{-1} - \gamma)\left[\frac{B_{t-1}}{P_{t-1}} - \bar{b} \right]$$

where B_t is nominal government debt, $\bar{z}$ is the steady-state level of transfers, and γ is the reaction coefficient of lump-sum taxes to the deviation of debt to the debt target, $\bar{b}$.

If the central bank follows the Taylor rule and responds aggressively to inflation ($\alpha > \beta$), the unique bounded solution of equation (1) for inflation is the inflation target. With monetary policy determining inflation, the expected evolution of real debt is given by equation (2). Because debt above target generates expectations of higher taxes in the passive fiscal policy regime, the debt is expected to return to steady state following a shock. Passive tax policy implies that fiscal adjustment must occur regardless of the reason why debt increases such as economic downturns or changes in household portfolio preferences, or central bank open-market operations. This is also the case in the Blanchard-Yaari model discussed in Section 3. Although we entertain periods of passive monetary policy (with interest rates at the zero lower bound) and active fiscal policy (with transfers not responding to debt), ultimately both reaction functions need to kick in to ensure a unique equilibrium with the central bank determining the inflation objective.

In the regime with passive monetary policy (e.g. a constant interest rate set by the central bank) and active fiscal policy (e.g. a constant tax rate set by the fiscal authorities) on the other hand, the price level will be determined by fiscal policy. This can be seen from the expression of the value of government debt obtained by imposing equilibrium on the government's flow constraint, taking conditional expectations, and "solving forward":

$$(3) \qquad \frac{B_t}{P_t} = E_t \sum_{j=1}^{\infty} \beta^j (\tau_{t+j} - z_{t+j})$$

The real value of the nominal debt has to be equal to the expected future primary surpluses. Substituting the active fiscal policy (a constant tax rate) into the forward-looking expression for debt and assuming that monetary policy sets the nominal interest rate on debt ($\overline{R}$), one can solve for the unique value of the price level:

$$(4) \qquad P_t = \frac{\overline{R} B_{t-1}}{\left(\frac{1}{1-\beta}\right)\overline{\tau} - E_t \sum_{j=1}^{\infty} \beta^j z_{t+j}}$$

In this environment changes in debt do not elicit any changes in expected taxes. As a result, at initial prices households feel wealthier and they try to shift their consumption patterns. Higher demand for goods drives up the price level and continues to do so until the wealth effect dissipates and households are content with their initial consumption plan. In this regime, the impact of monetary policy changes dramatically. When the central bank chooses a higher interest rate, the effect is to raise inflation in the next period. The higher interest rate payments increase income, consumption and the price level. As discussed in Leeper and Walker (2011), more realistic adjustment patterns may take place if government debt is long-term. In that case, the maturity composition of existing government debt may determine the pattern of inflation following a fiscal shock. However, also in this case the value of the long-term government bond will necessarily go down. Interestingly, Cochrane (2011) shows that in such a case, buying long-term debt for short-term debt will increase inflation now relative to later.

As discussed in Section 2, currently the prices of long-term bond yields in the advanced economies like the United States, the United Kingdom and Germany are still very low and medium to long-term inflation expectations are stable. This raises the question under what circumstances fiscal policy may undermine monetary control of inflation. A negative shock to government revenues due to a recession like discussed in Section 3 may undermine the soundness of fiscal policy, lead to a higher fiscal deficit and an unsustainable accumulation of government debt, while at the same reduce short-term nominal interest rates to zero. The lower bound on interest rates will in that case imply a passive monetary policy, while the accumulation of government debt may become unsustainable as the fiscal limit is reached.[11] If such a situation leads to a perceived probability of a switch to an active fiscal policy/passive monetary policy regime, this by itself will have an impact on inflation and inflation expectations. Leeper and Walker (2011) show that in this case, the economy will not exhibit Ricardian equivalence, but the quantitative effects will depend on how large the shock to public finances and the fiscal space is. In such a situation, higher expected deficits may start reducing the value of debt because they reduce the backing and therefore the value of government liabilities and monetary policy may loose control of inflation. The current relatively benign long-term interest rates in the largest advanced economies may not be of much comfort to the extent that the switch to an active fiscal/passive monetary policy regime may occur quite abruptly as shown in Bi, Leeper and Leith (2011).

4.2. Evidence on the link between public debt and inflation

The theoretical possibility of a link between public debt and inflation is clear. What is the historical evidence of such a link? A commonly held view about the origins of inflation and excessive money growth is that it results from fiscal imbalances. In this section we review two cross-country papers that have investigated this link and present some suggestive evidence.

In their study of historical episodes of high inflation (greater than 100%), Fischer, Sahay and Vegh (2002) find that on average a 10 percentage point reduction in the fiscal balance is associated with a 1.5 percentage increase in seignorage revenues (as percent of GDP). This relationship is, however, much stronger for high-inflation countries, where a 10 percentage point reduction is associated with a 6.5 percentage increase in seignorage. In those countries, a reduction in the fiscal balance by 1 percent of GDP leads to an increase in the inflation rate by 4.2 percent, but no obvious

[11] Uhlig and Trabandt (2011, 2012) discuss fiscal limits in the US and across countries in Europe on the basis of estimates of Laffer curves.

long or short-run relationship between inflation and fiscal balance is found for the low-inflation countries.

A recent study that directly addresses the link between public debt and inflation in a sample of 71 advanced and developing countries over the period 1963-2004 is Kwon, McFarlane and Robinson (2009). Their regression results show a strong and stable positive effect of debt growth on inflation in developing and non-major advanced economies. The coefficient for public debt is nearly 0.2 for the short term and 0.25 for the long term, which implies that a 1 percent increase in public debt leads to a 0.2 percentage point increase in inflation. The short and long-term coefficients are lower than those of money growth, but are significant at the 5 percent level and rise to 0.3 and 0.5, respectively, for a subset of 25 indebted developing countries. The existence of the strong debt-inflation linkage, after controlling for money growth, suggests that the link between public debt and inflation may go beyond the transmission through money growth. In contrast, in 13 major advanced economies, none of the explanatory variables, except lagged inflation, show significant short-term associations with inflation. This finding is somewhat dependent on the empirical methodology used. For example, using mean group estimators, they do find evidence of an effect of the growth in public debt on inflation also in advanced economies, but the effect is again much stronger in less-developed countries with high foreign indebtedness. This evidence shows that the policy regime is of great importance.

In the rest of this section we present some suggestive evidence about the bilateral relationship between public debt and inflation using a database collected by Fratszcher, Mehl and Vansteenkiste (2011), who focus on the link between public debt and currency crashes. The database covers 17 advanced countries, six of which contain data starting in the 1910s and eleven of which typically start in the 1950s.[12] Broadly speaking there have been two waves of big increases in public debt in those countries before the current rise in debt. The first big wave is mostly due to the second world war. The second wave occurred in response to the oil price crises in the 1970s and the productivity slowdown.

---

[12] The first sample inc

{Insert Chart 18}

Chart 18 plots the cross correlation between public debt and inflation in two partly overlapping samples covering the two waves. The first sample is from 1910 to 1970 and covers six advanced countries. The second sample is from 1950 to 2009 and covers 15 advanced countries. The charts show that there is a positive correlation between current and lagged public debt and inflation in the earlier sample period, suggesting that following the build-up of nominal debt during the second world war, inflation was one way of reducing the nominal debt. This is consistent with the discussion in Aizenham and Marion (2010) for the United States and also complementary to the findings in Reinhart and Sbrancia (2011). The latter finds that financial repression (i.e. negative real interest rates) has been one common way in which public debt has been dealt with in the past. In contrast, in the post second world war sample, there is a negative correlation between current and lagged debt and inflation. These results are confirmed by the impulse responses from a simple bilateral VAR(2) in Chart 19 and 20.

{Insert Chart 19 and 20}

Without a structural model, it is difficult to interpret these reduced-form findings. However, both the cross-country differences and the differences over time suggest that the institutional framework and the credibility of the policy regime are key in explaining the presence or absence of such a link. In particular, in the past advanced economies may have had a greater capacity to adjust taxes and spending to contain and reduce increases in public debt. Secondly, the monetary policy frameworks established in the 1980s in response to the great inflation experience may have contributed to a stabilisation of inflation and inflation expectations. Indeed, Leeper, Chung and Davig (2007) find that most of the post-1980s period is characterised by active monetary policy.

5. Conclusions

[to be done]

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Chart 1

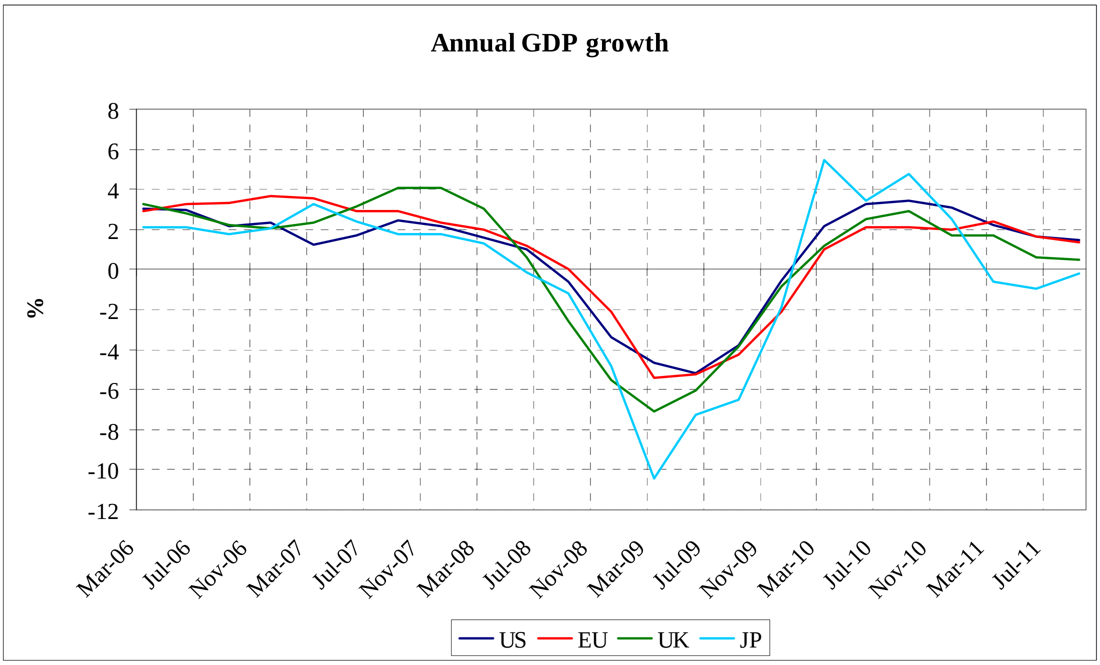


Chart 2

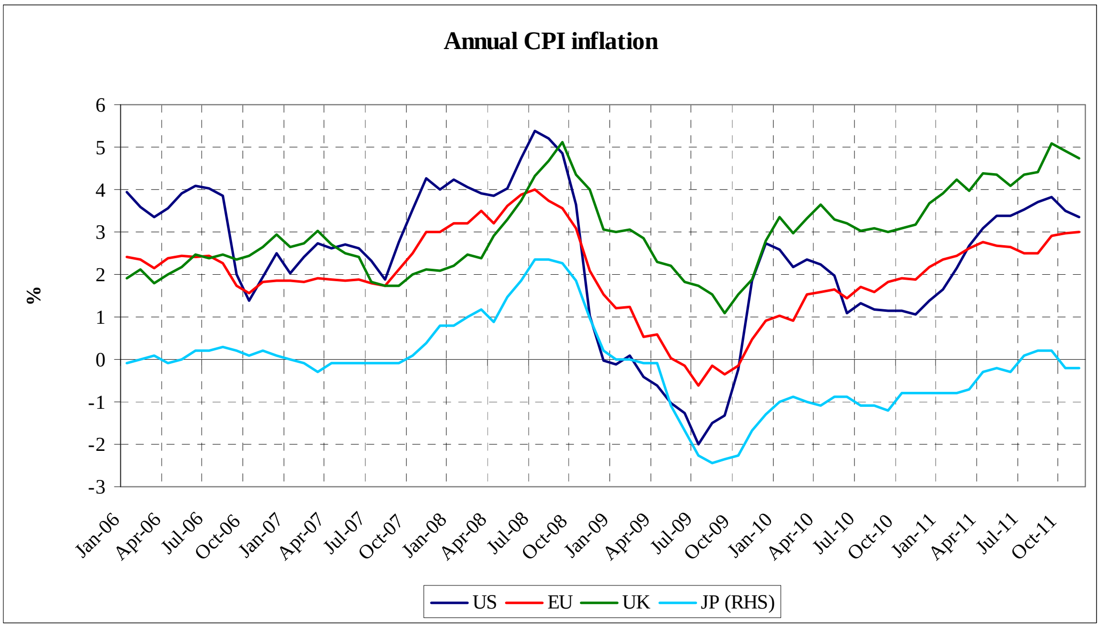
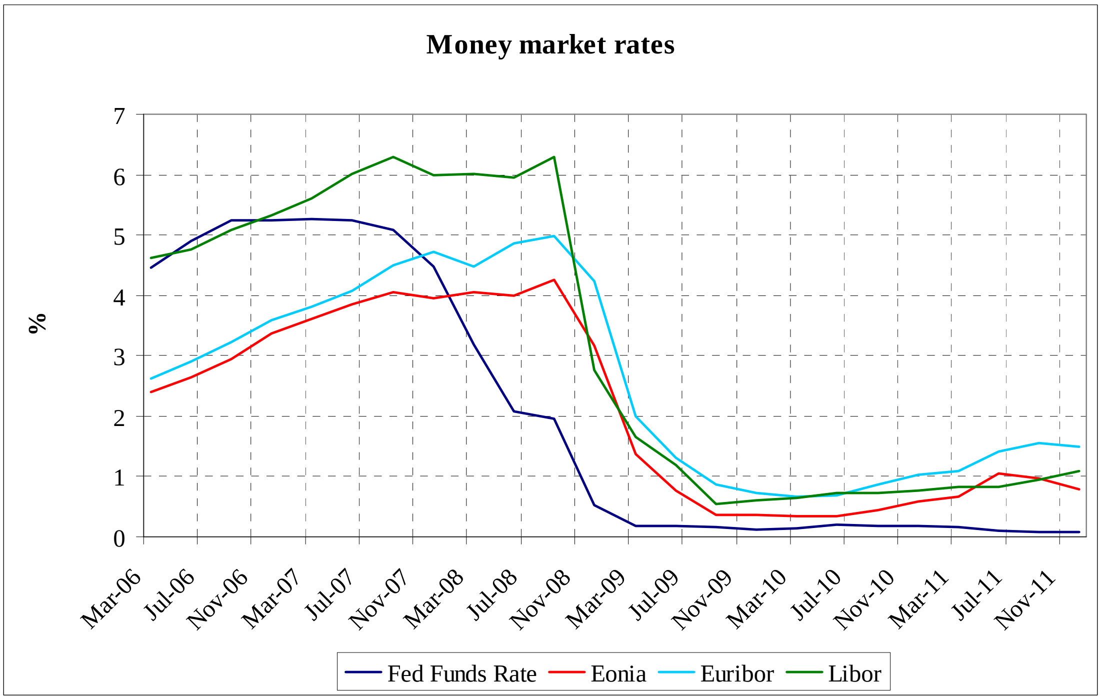


Chart 3



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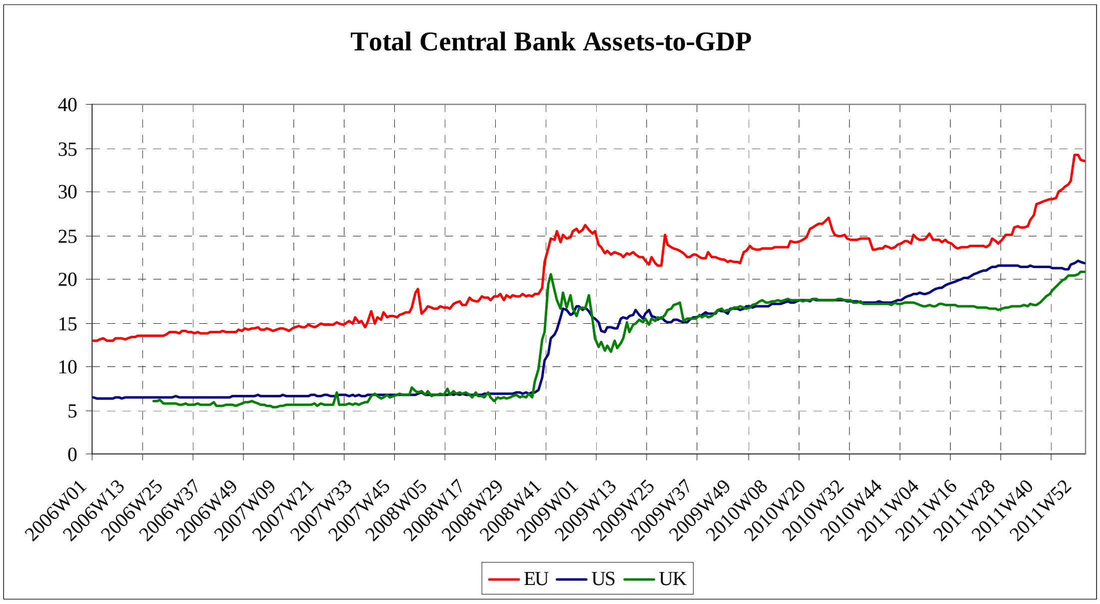
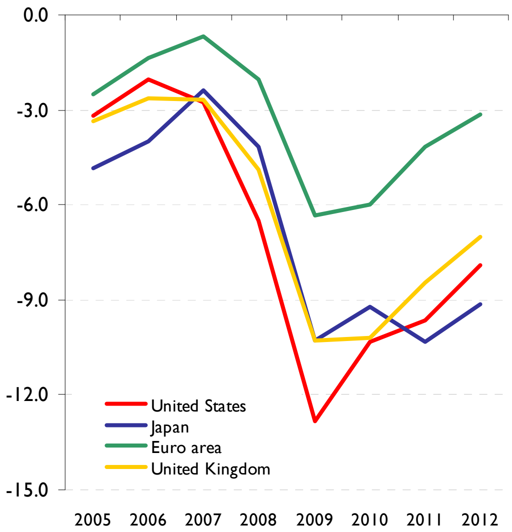


Chart 5

General government budget balance
(2005-2012, percentage of GDP)



General government structural balance
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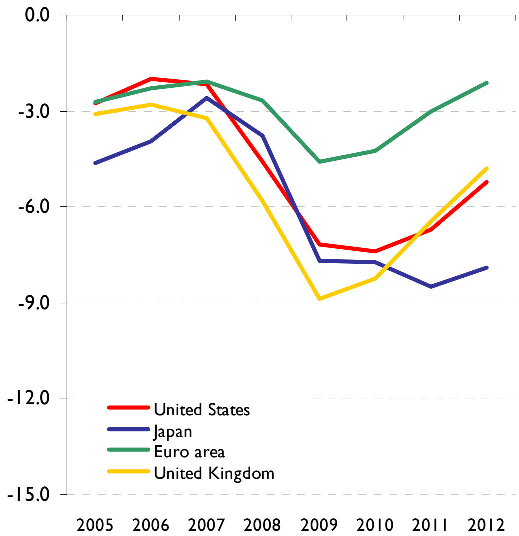
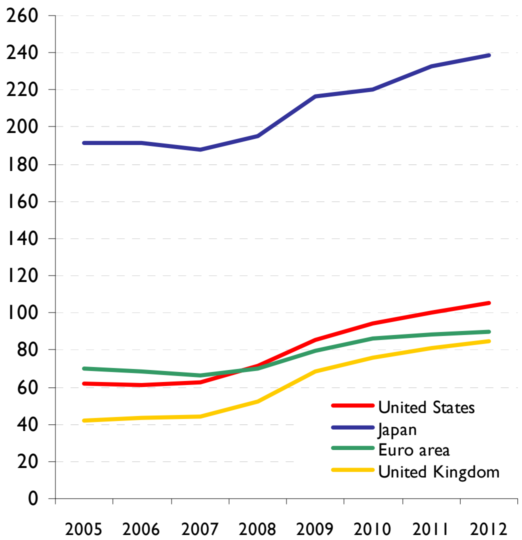


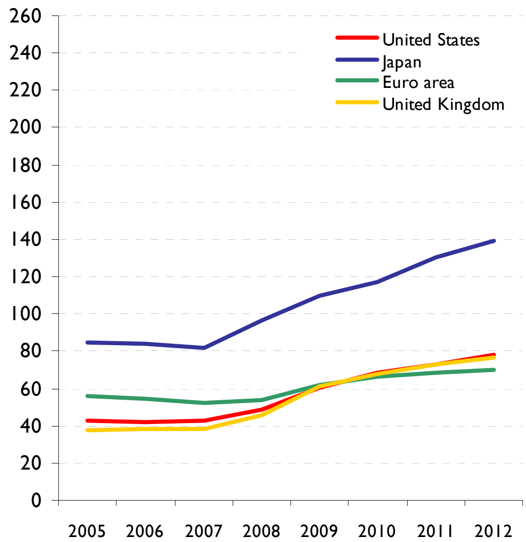
Source: IMF World Economic Outlook September 2011

Chart 6

General government gross debt
(2005-2012, percentage of GDP)



General government net debt
(2005-2012, percentage of GDP)



Source: IMF World Economic Outlook September 2011

Chart 7

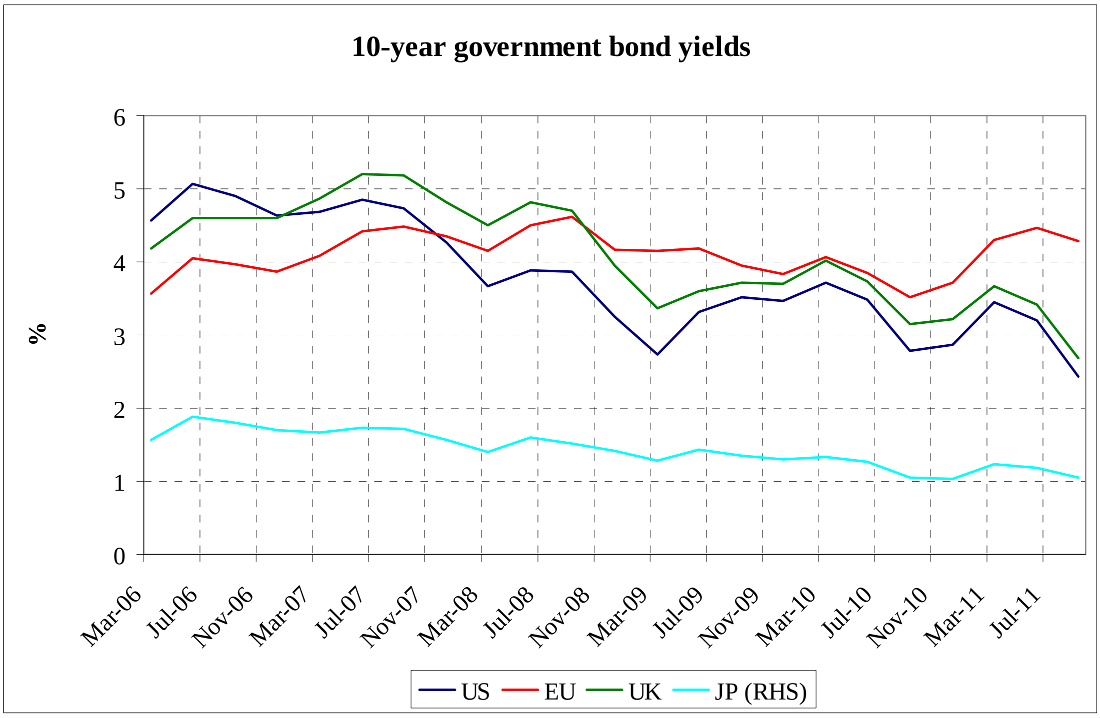
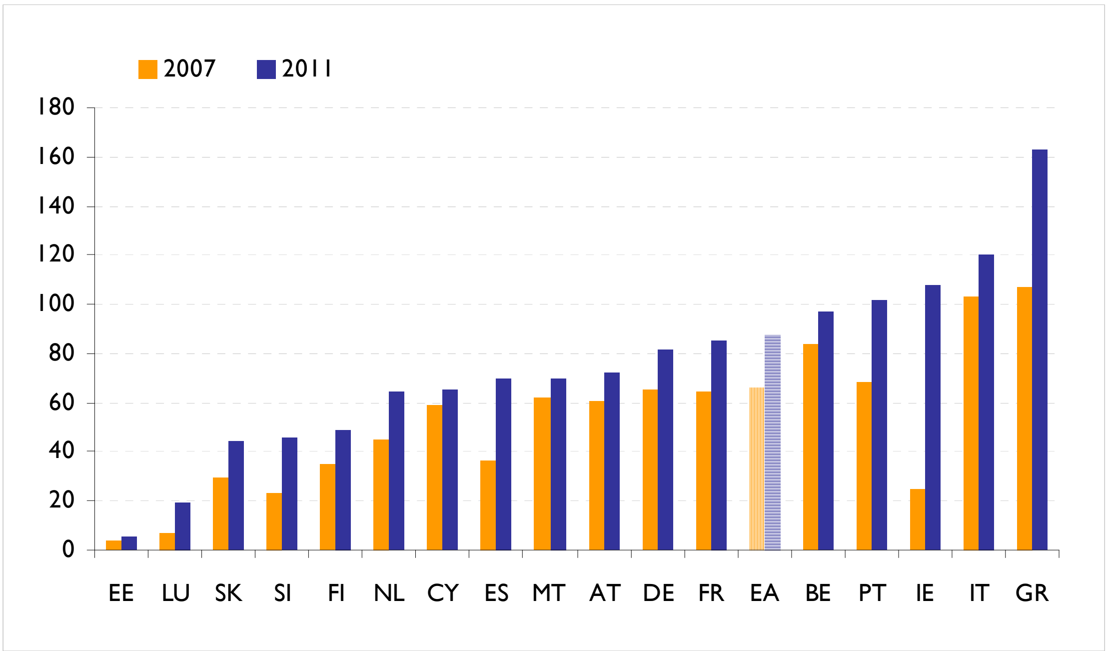


Chart 8

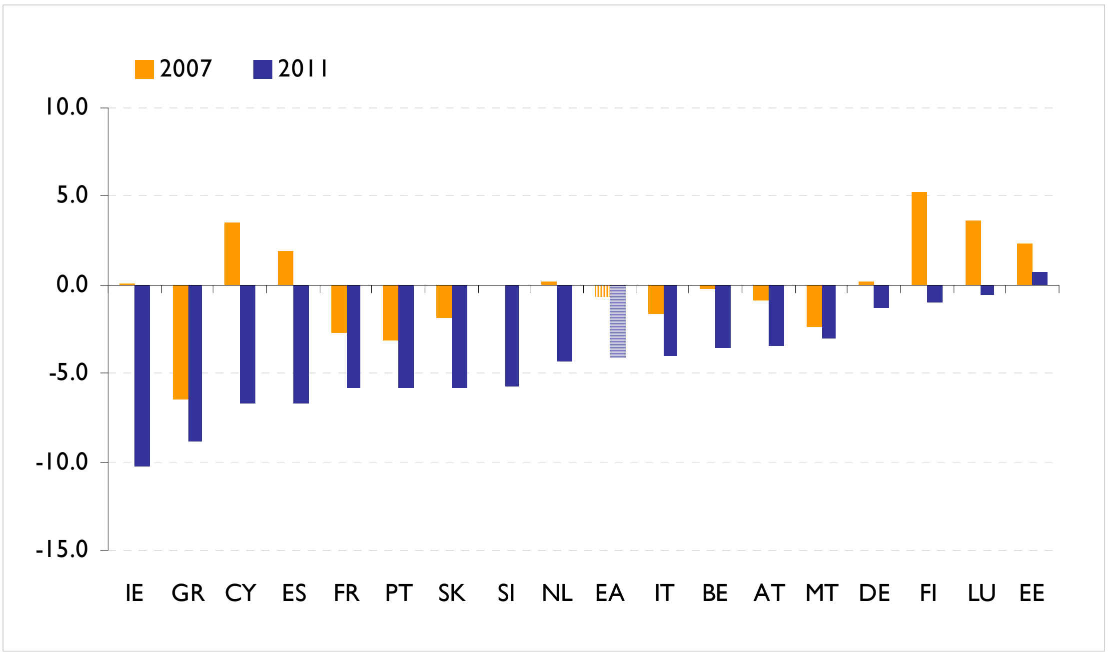
General government gross debt (% of GDP)



Source: European Commission autumn 2011 economic forecast.

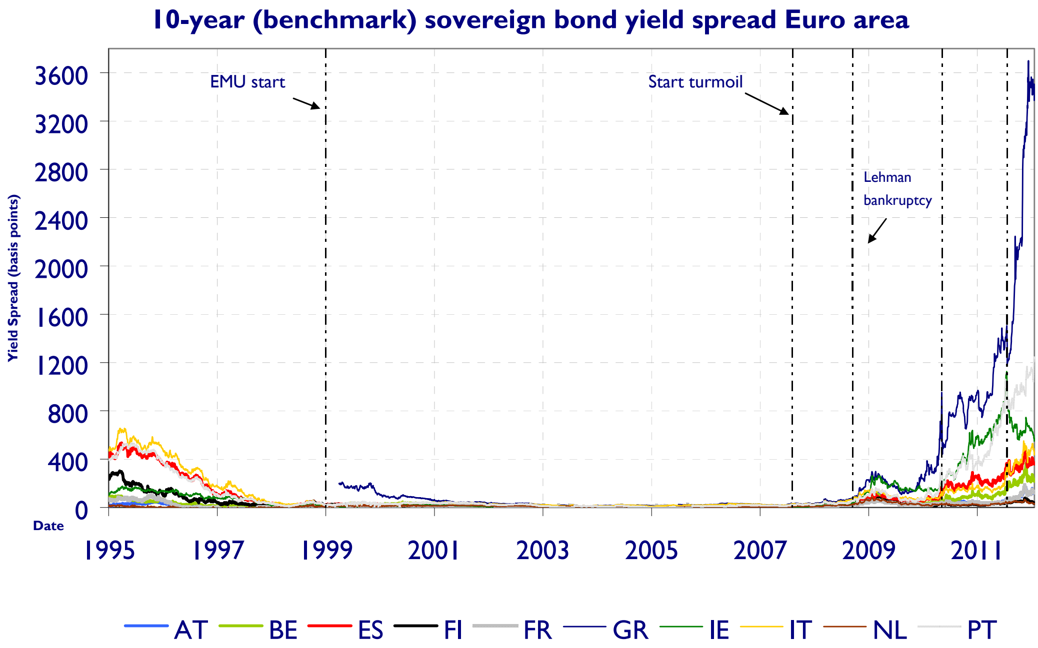
Chart 9

General government budget deficit (% of GDP)



Source: European Commission autumn 2011 economic forecast.

Chart 10



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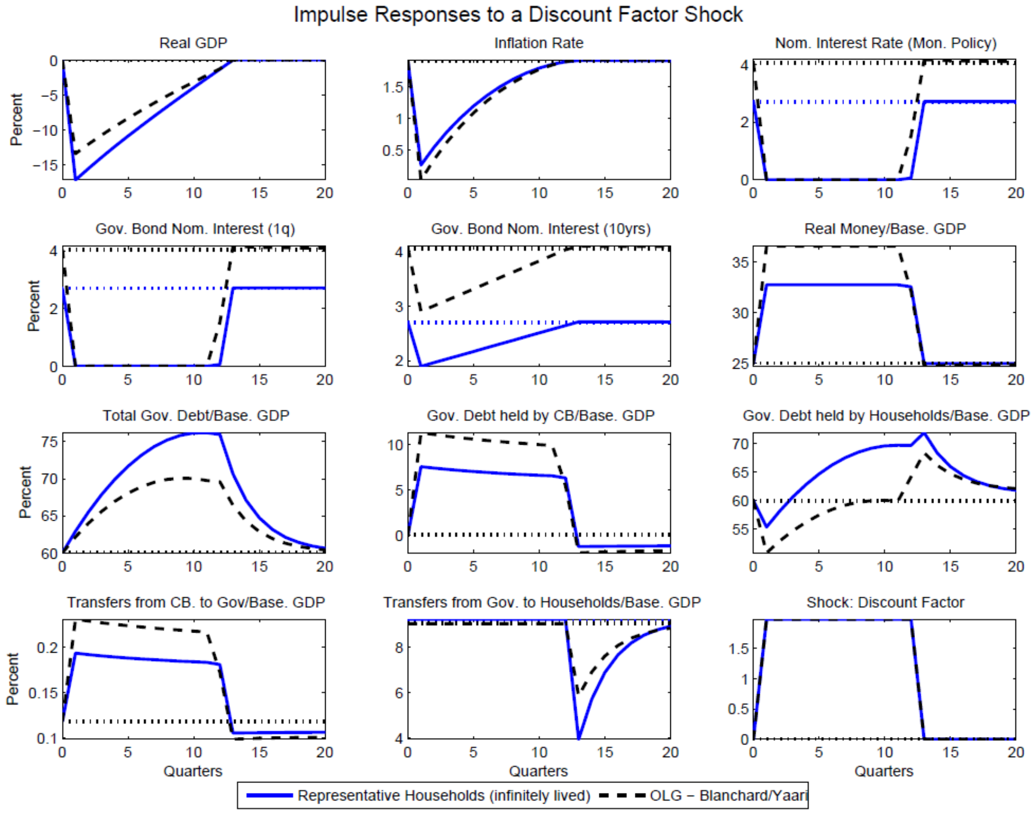

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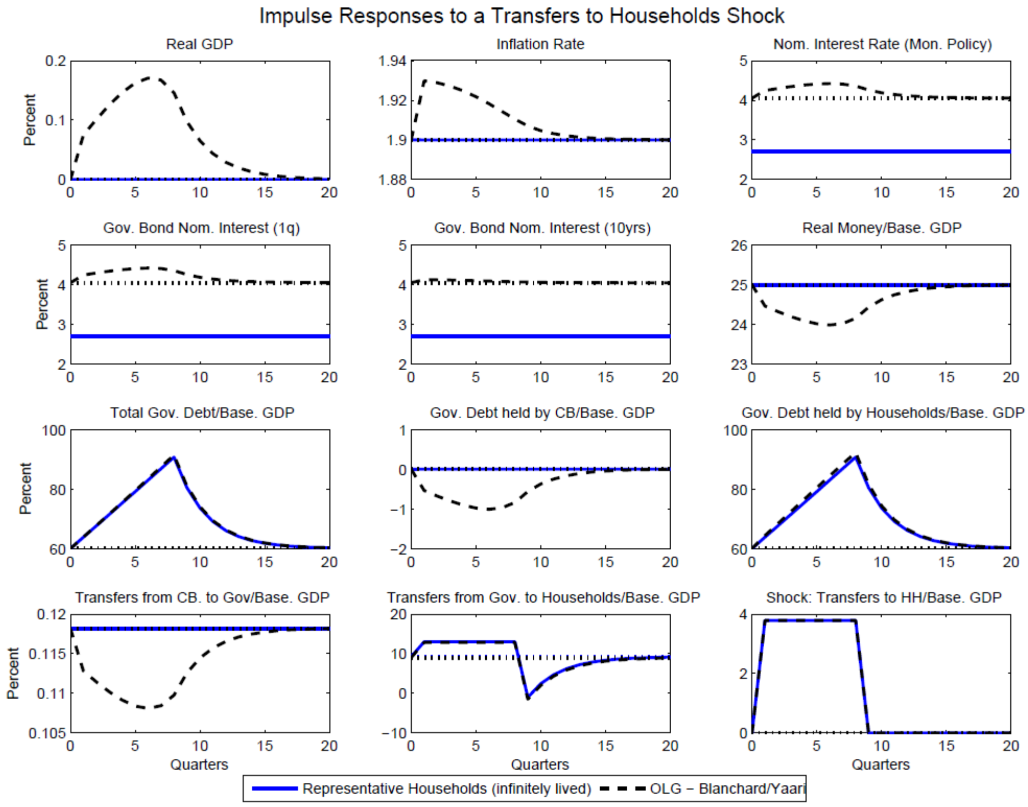
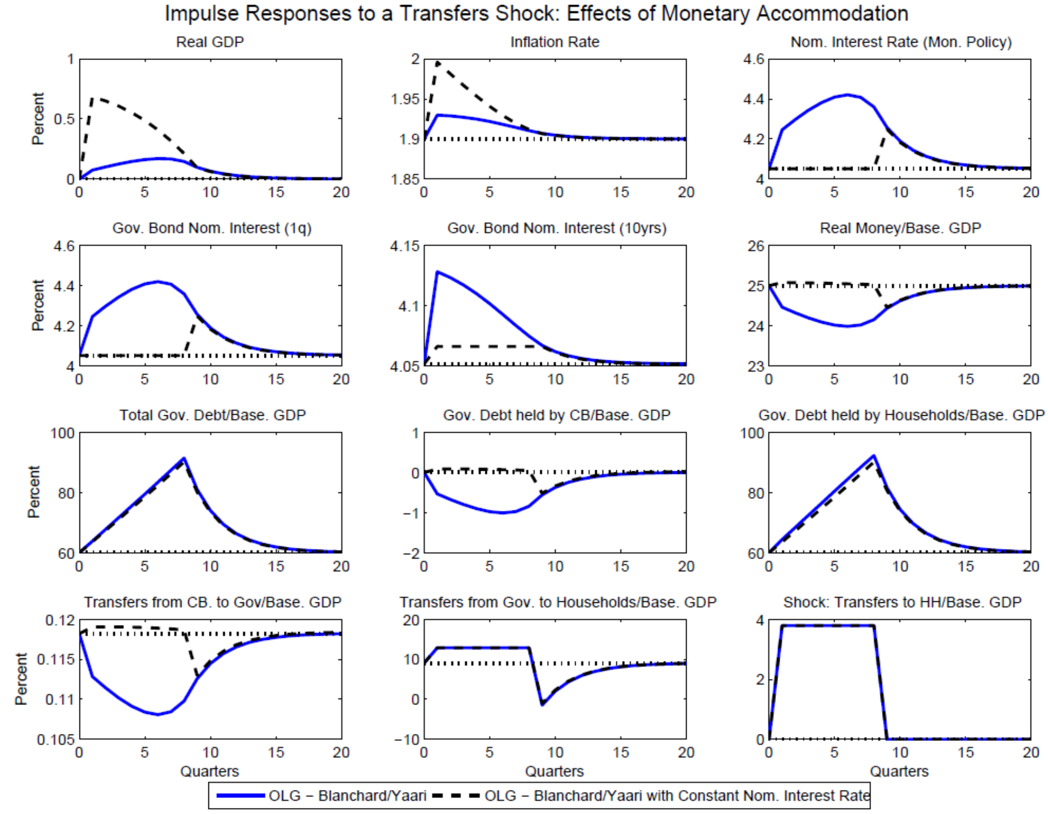


Chart 13



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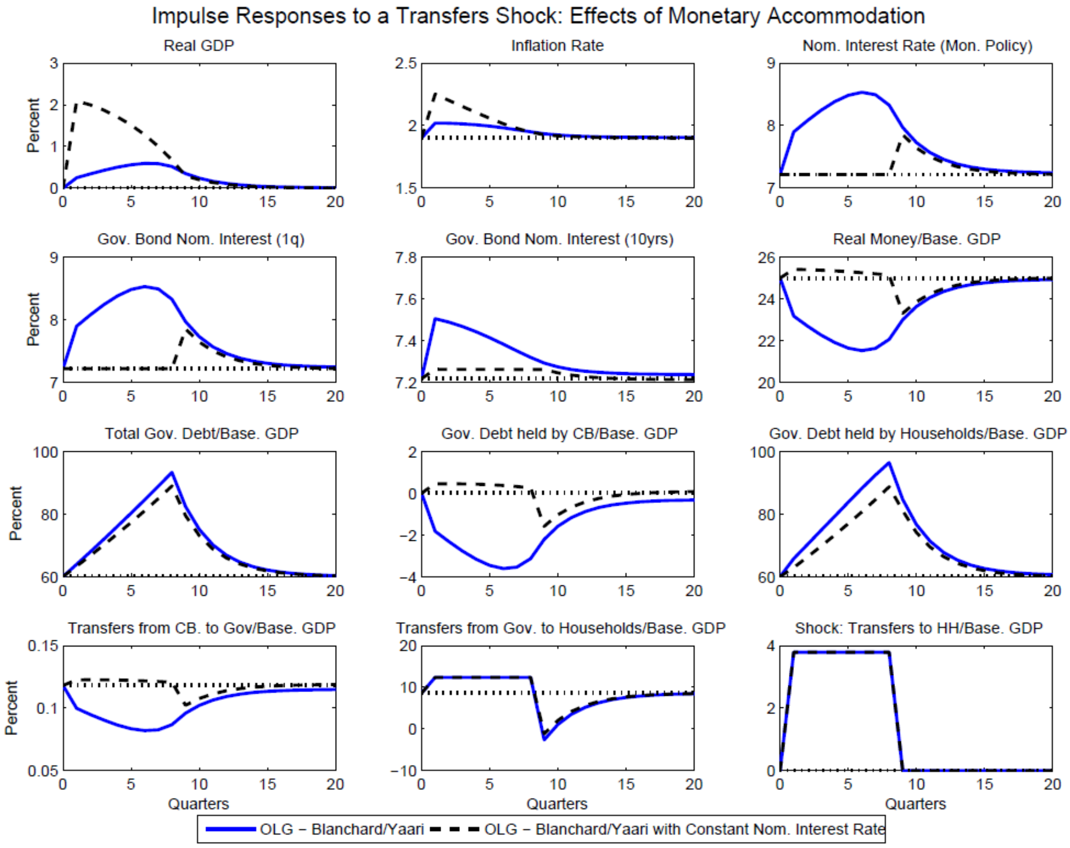


Chart 15

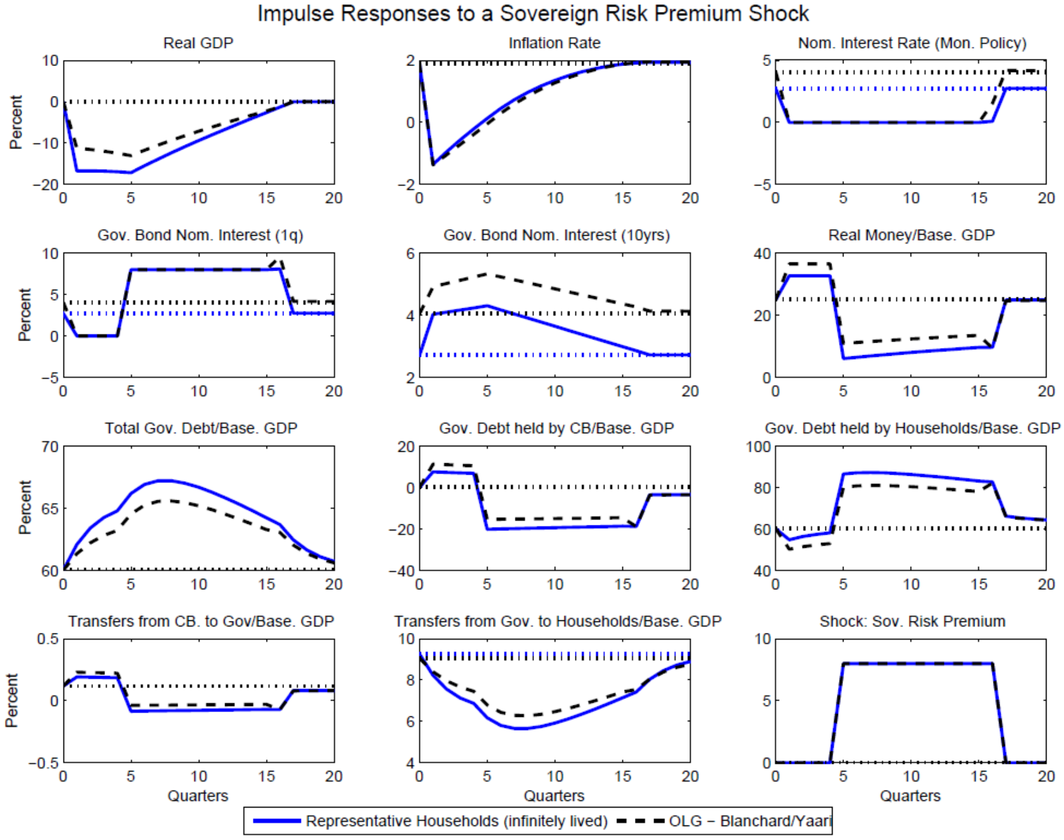


Chart 16

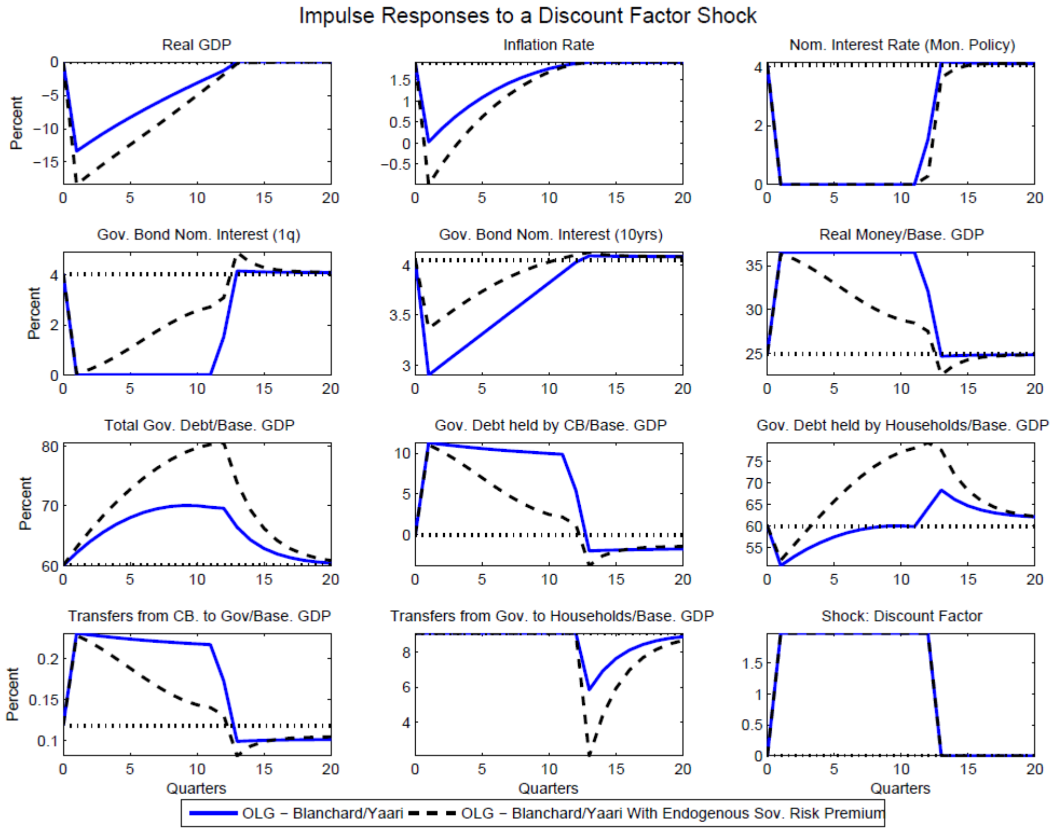


Chart 17

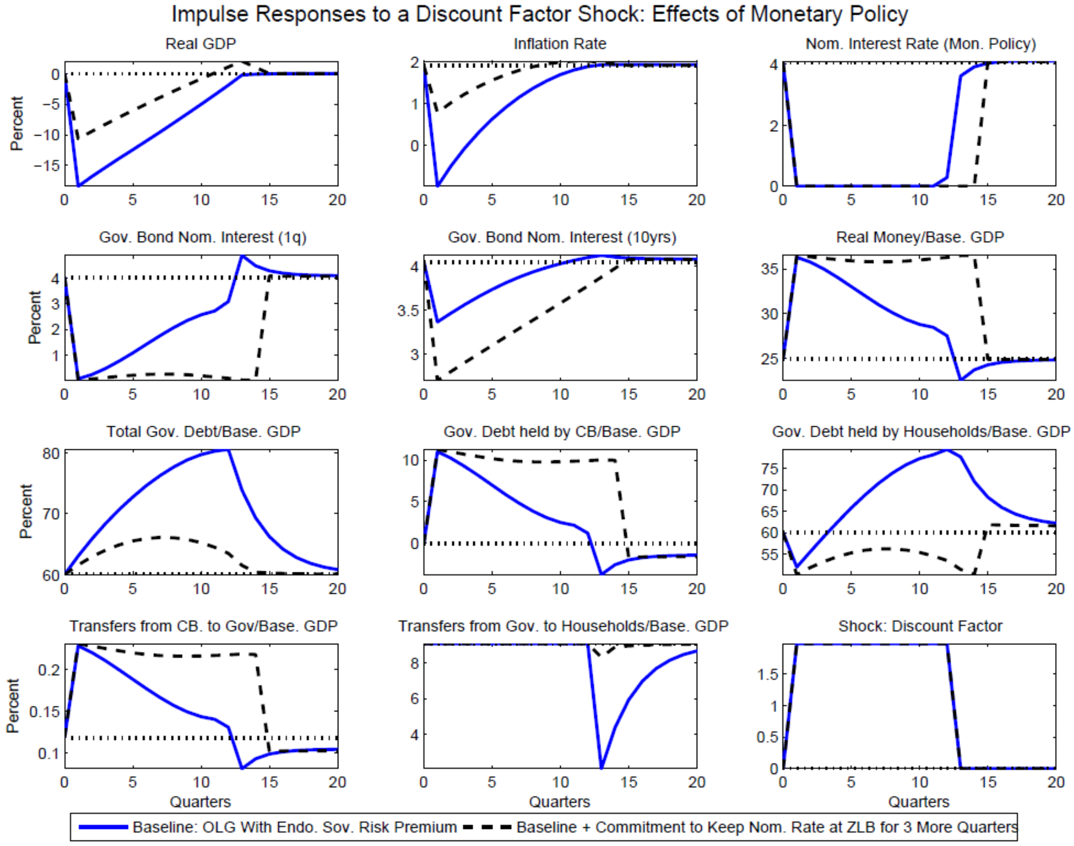


Chart 18

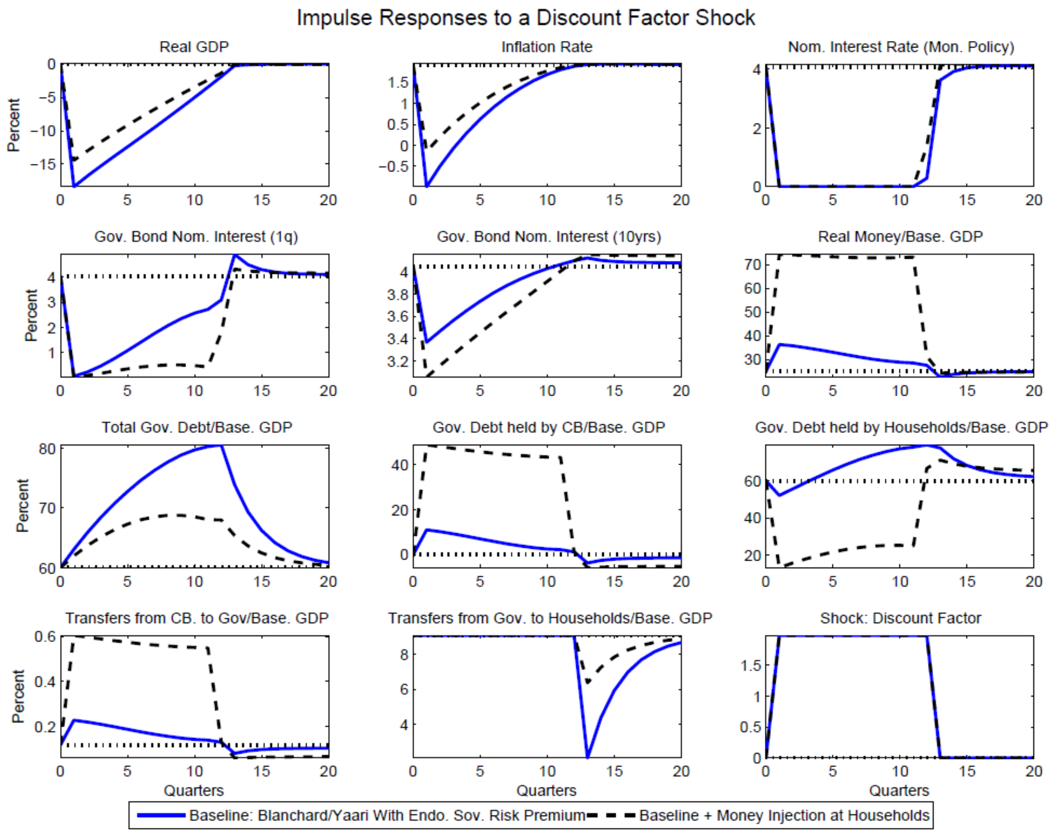
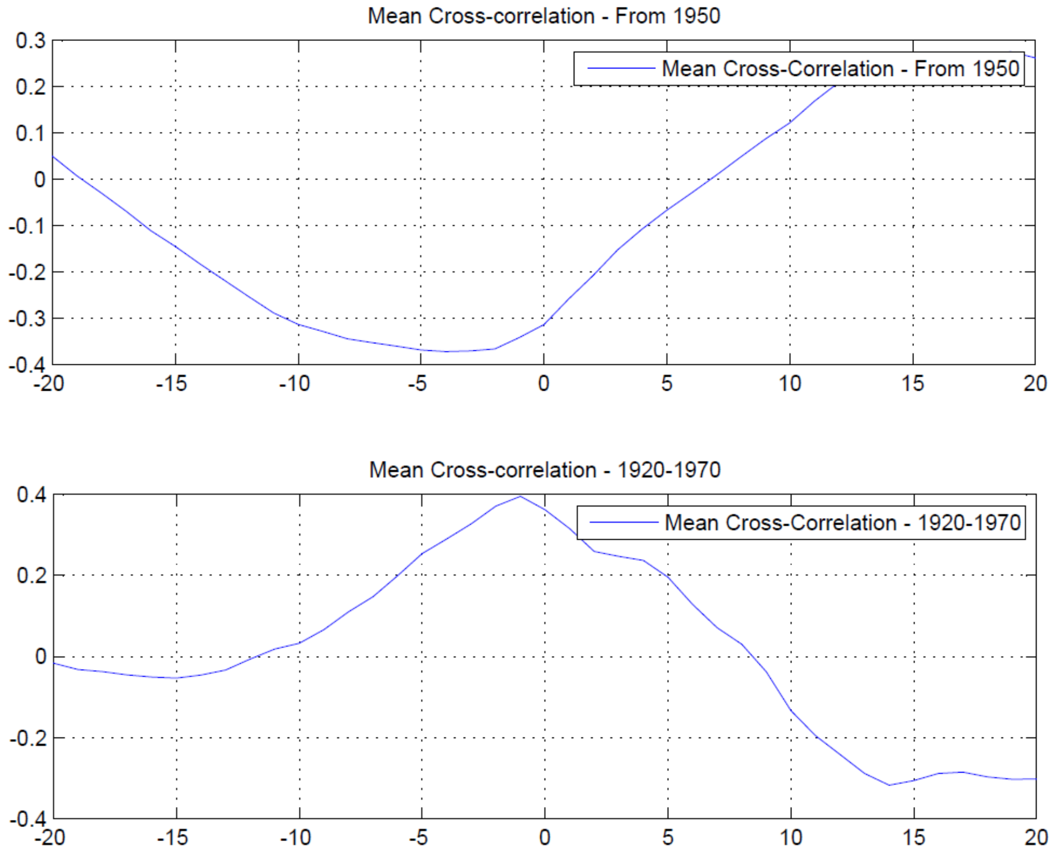


Chart 19

Cross-correlation between debt-to-GDP(t-k) and inflation(t)

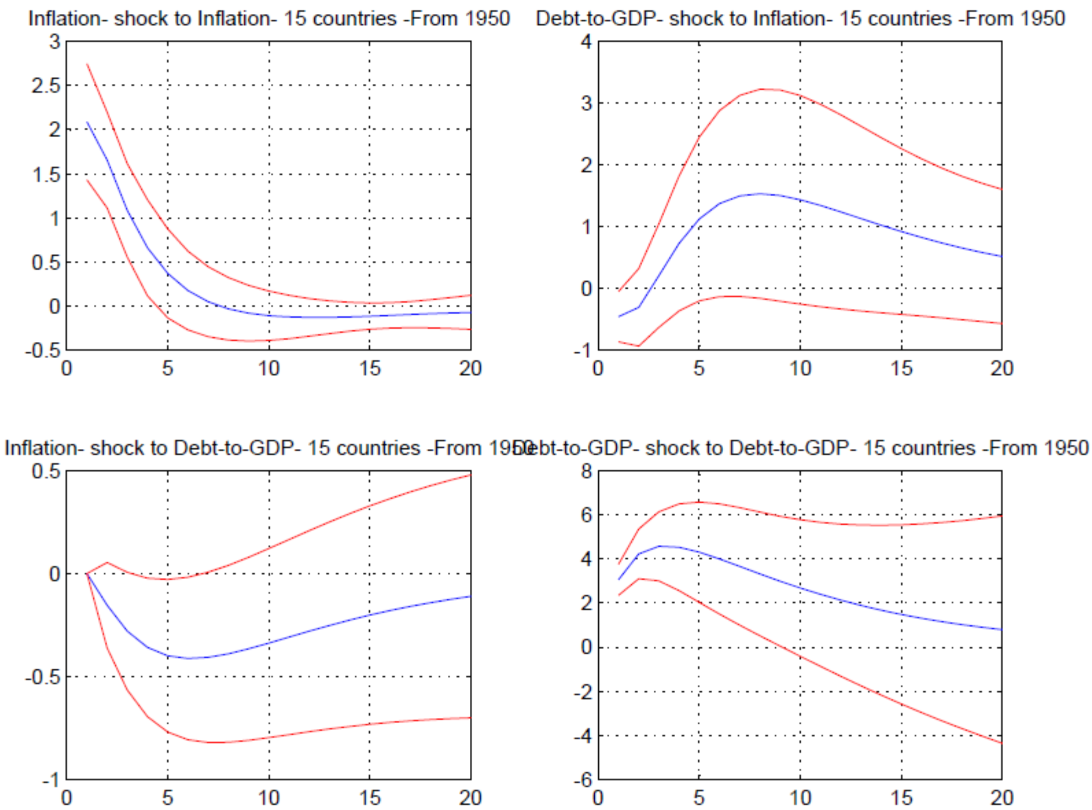


Source: Dataset from Fratszcher, Mehl and Vansteenkiste (2011); Annual data: the sample in the upper panel contains 15 advanced economies; the sample in the lower panel contains six advanced economies. Mean group estimates.

Chart 20

Impulse response to an inflation and debt-to-GDP shock

1950s-2009: 15 countries

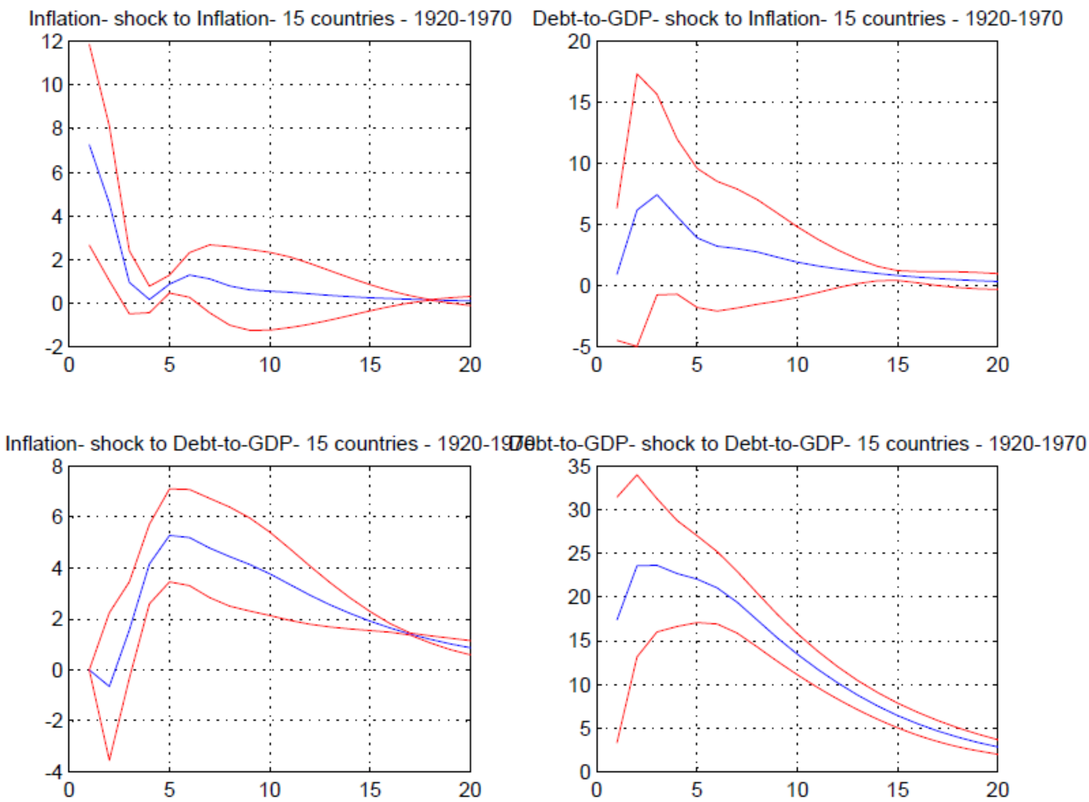


Source: Dataset from Fratszcher, Mehl and Vansteenkiste (2011); annual bilateral VAR(2) in inflation and debt-to-GDP ratio. Choleski decomposition with debt-to-GDP ordered last. Mean group estimates.

Chart 21

Impulse response to an inflation and debt-to-GDP shock

1920-1970: 5 countries



Source: Dataset from Fratszcher, Mehl and Vansteenkiste (2011); annual bilateral VAR(2) in inflation and debt-to-GDP ratio. Choleski decomposition with debt-to-GDP ordered last. Mean group estimates.

Appendix

1 Blanchard-Yaari Model with Sticky Prices

- The following setup is similar to the one in e.g. Devereux (2011).
- Blanchard (1985)-Yaari (1965) perpetual youth in discrete time
- Households die with probability $(1-\delta)$ each period
- Each period, newborn generation j represents fraction $(1-\delta)$ of total population
- Full annuities market (perfectly competitive life insurance industry); borrowers pay a premium to cover their posthumous debt; savers get a premium on lending to cover their unintended bequests
- Full annuity markets imply that rates of return are grossed up to cover the probability of death:

$$\delta(1+i_t)/\delta + (1-\delta) * 0 = 1 + i_t$$

- Households have no bequest motive. They sell contingent claims on their assets to perfectly competitive insurance companies. Assets from the $(1-\delta)$ exiting households are transferred to all non-exiting and newborn households. Hence, each surviving generation receives a premium payment, per unit of asset, of

$$\frac{\overbrace{1-\delta}^{\text{assets collected from dying households}}}{\underbrace{\delta}_{\text{number of surviving housholds assets are allocated to}}}$$

- Hence, the gross return on the insurance contract is

$$1 + (1-\delta)/\delta = 1/\delta > 1$$

which will the factor multiplying asset income per household.

- Log preferences needed for linearity in wealth and therefore ensure aggregation

2 Households

2.1 Optimality conditions

Households maximize

$$\max_{c_t^j, M_t^j, B_t^{H,j}, n_t^j} E_0 \sum_{t=0}^{\infty} (\beta\delta)^t \, \sigma_{t-1} \left[\log c_t^j - \frac{\nu_t}{2} \left(\frac{\overline{M^j}}{P} - \frac{M_t^j}{P_t} \right)^2 + A \log(1-n_t^j) \right]$$

subject to

$$P_t c_t^j + \frac{B_t^{H,j}}{R_t^{gov}} + M_t^j = \underbrace{(1-\tau_t) W_t n_t^j + \Theta_t^j + TR_t^j}_{\Omega_t^j} + \frac{1}{\delta} \underbrace{\left(B_{t-1}^{H,j} + M_{t-1}^j \right)}_{\Psi_t^j}$$

where $B_t^{H,j}$ denotes government bonds held by households of generation j. We assume a competitive labor market. The common nominal wage is denoted by W_t. Further, Θ_t^j are the share of profits of intermediate goods producers that go to generation j. Moreover, σ_{t-1} is a shock to utility, realized in the previous period. In equilibrium, the ratio of $\frac{\sigma_t}{\sigma_{t-1}}$ will the a shifter of the discount factor β in the Euler equation. That is, a positive realization of $\frac{\sigma_t}{\sigma_{t-1}}$ will induce a rise in the effective discount factor so that households want to save more. This will trigger a fall in consumption today and lead to a recession possibly implying a binding ZLB. Note that $R_t^{gov} = \gamma_t R_t$ where γ_t denotes a shock that drives a wedge between the nominal interest rate controlled by the central bank, R_t, and the nominal interest rate paid on government debt, R_t^{gov}. In other words, an increase of γ_t leads to a fall of the price of government debt which we will interpret later as an increase in sovereign risk. Finally, TR_t^j are lump-sum transfers from the government to generation j households.

Optimality:

$$\begin{aligned}
\frac{M_t^j}{P_t} &= \frac{\overline{M^j}}{P} - \left[\frac{R_t^{gov}-1}{R_t^{gov}}\right]\frac{1}{\nu_t c_t^j} \\
\frac{Ac_t^j}{1-n_t^j} &= (1-\tau_t)\frac{W_t}{P_t} \\
1 &= \beta E_t \frac{\sigma_t}{\sigma_{t-1}}\frac{c_t^j}{c_{t+1}^j}\frac{R_t^{gov}}{\Pi_{t+1}}
\end{aligned}$$

2.2 Aggregation

2.2.1 Household budget constraint

$$P_t c_t + \frac{B_t^H}{R_t^{gov}} + M_t = (1-\tau_t) W_t n_t + \Theta_t + TR_t + B_{t-1}^H + M_{t-1}$$

2.2.2 Labor/leisure trade-off

$$\frac{Ac_t}{1-n_t} = (1-\tau_t)\frac{W_t}{P_t}$$

2.2.3 Approximate money demand

$$\frac{M_t}{P_t} = \frac{\overline{M}}{P} - \left[\frac{R_t^{gov}-1}{R_t^{gov}}\right]\frac{1}{\nu_t c_t}$$

2.2.4 Euler Equation

$$\beta\frac{\sigma_t}{\sigma_{t-1}} c_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1-\delta}{\delta\mu_{t+1}\Pi_{t+1}}\left[\frac{B_t^H}{P_t} + \frac{M_t}{P_t}\right] + c_{t+1}$$

3 Final Goods Firms

Production:

$$y_t = \left(\int y_{t,i}^{\frac{1}{\omega}}\, di\right)^{\omega}, \omega > 1$$

Profit maximization:

$$\max_{y_{t,i}} \; P_t \left(\int y_{t,i}^{\frac{1}{\omega}}\, di\right)^{\omega} - \int P_{t,i} y_{t,i} di$$

Optimal input demand:

$$y_{t,i} = \left(\frac{P_t}{P_{t,i}}\right)^{\frac{\omega}{\omega-1}} y_t$$

Input price index:

$$P_t = \left(\int P_{t,i}^{\frac{1}{1-\omega}}\, di\right)^{1-\omega}$$

Aggregate equilibrium profits:

$$\int \Pi_t^f \ di = \Pi_t^f = 0$$

4 Intermediate Good Firms

4.1 Flexible Prices:

Production:

$$y_{t,i} = n_{t,i}$$

Profit maximization:

$$\max_{P_{t,i}} \ P_{t,i} y_{t,i} - (1 - \chi_t) W_t n_{t,i}$$

subject to

$$\begin{aligned} y_{t,i} &= \left(\frac{P_t}{P_{t,i}}\right)^{\frac{\omega}{\omega-1}} y_t \\ y_{t,i} &= n_{t,i} \end{aligned}$$

where χ_t denotes a subsidy. Note that marginal costs are:

$$MC_t = (1 - \chi_t) W_t$$

Optimal price:

$$P_{t,i} = \omega MC_t = \omega (1 - \chi_t) W_t$$

Hence, in equilibrium, all firms set the same price $P_{t,i} = P_t$. Accordingly, $y_{t,i} = y_t$, $n_{t,i} = n_t$ and $y_t = n_t$. Aggregate equilibrium profits:

$$\int \Pi_t^i \ di = \Pi_t^I = (\omega - 1)(1 - \chi_t) W_t y_t$$

4.2 Sticky Prices:

Profit maximization:

$$\max_{\tilde{P}_t} \ E_0 \sum_{j=0}^{\infty} (\delta \xi_p \beta)^j \Lambda_{t+j} \left[\Pi^j \tilde{P}_t y_{t+j,i} - MC_{t+j} y_{t+j,i} \right]$$

subject to

$$y_{t+j,i} = \left(\frac{P_{t+j}}{\Pi^j \tilde{P}_t}\right)^{\frac{\omega}{\omega-1}} y_{t+j}$$

Optimality implies

$$E_t \sum_{j=0}^{\infty} (\delta \xi_p \beta)^j \Lambda_{t+j} y_{t+j} P_{t+j} \left[(X_{t,j} \tilde{p}_t)^{1-\frac{\omega}{\omega-1}} - \omega mc_{t+j} (X_{t,j} \tilde{p}_t)^{-\frac{\omega}{\omega-1}} \right] = 0$$

with

$$mc_{t+j} = \frac{MC_{t+j}}{P_{t+j}}$$

Or

$$E_t \sum_{j=0}^{\infty} (\delta \xi_p \beta)^j \Lambda_{t+j} P_{t+j} y_{t+j} (X_{t,j})^{-\frac{\omega}{\omega-1}} [X_{t,j} \tilde{p}_t - \omega mc_{t+j}] = 0$$

Or

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} \left(\delta \xi_p \beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{\omega}{\omega-1}} \omega mc_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\delta \xi_p \beta\right)^j \lambda_{t+j} y_{t+j} X_{t,j}^{-\frac{1}{\omega-1}}} = \frac{K_t}{F_t}$$

where

$$\lambda_{t+j} = \Lambda_{t+j} P_{t+j}$$

and

$$K_t = \lambda_t y_t \omega mc_t + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{\omega}{\omega-1}} K_{t+1}$$

and

$$F_t = \lambda_t y_t + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{1}{\omega-1}} F_{t+1}$$

Further, the aggregate price index equation can be rewritten as follows:

$$\tilde{p}_t = \left[\frac{1 - \xi_p \left(\frac{\Pi}{\Pi_t}\right)^{\frac{1}{1-\omega}}}{1 - \xi_p}\right]^{1-\omega}$$

Measure of price dispersion:

$$\mathring{p}_{t-1} = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\Pi}{\Pi_t}\right)^{\frac{1}{1-\omega}}}{1 - \xi_p}\right)^{\omega} + \xi_p \left(\frac{\Pi}{\Pi_t} \mathring{p}_{t-1}\right)^{\frac{\omega}{1-\omega}}\right]^{\frac{1-\omega}{\omega}}$$

Aggregate output:

$$y_t = n_t \mathring{p}_t^{\frac{\omega}{\omega-1}}$$

5 Government

Budget constraint:

$$B_{t-1}^G + TR_t + \chi_t W_t \int y_{t,i} = \frac{B_t^G}{R_t^{gov}} + \tau_t W_t \int n_{t,i} + S_t$$

where B_t^G denotes total debt issued by the government. Further, S_t is a transfer from the central bank.

6 Central Bank

Budget constraint:

$$\frac{B_t^M}{R_t^{gov}} + S_t = B_{t-1}^M + M_t - M_{t-1}$$

where B_t^M denotes sovereign debt held by the central bank. S_t denotes a transfer from the central bank to the government. Further, we assume that the central bank sets the nominal interest rates according to:

$$R_t = R + \phi_\pi \left(\Pi_t - \Pi\right) + \phi_y \left(\frac{y_t}{y} - 1\right)$$

7 Equilibrium

Government bond market clearing:

$$B_t^G = B_t^M + B_t^H$$

Aggregate budget constraint:

$$c_t = \frac{W_t}{P_t} n_t - \chi_t \frac{W_t}{P_t} \int y_{t,i} + \frac{\Theta_t}{P_t}$$

Aggregate intermediate firms profits:

$$\frac{\Theta_t}{P_t} = y_t - (1 - \chi_t) \frac{W_t}{P_t} n_t$$

Hence, the resource constaint reads

$$c_t = y_t$$

8 Equilibrium

$$\text{Bond Market Clearing} \quad : \quad b_t^G = b_t^M + b_t^H$$

$$\text{Central Bank Budget} \quad : \quad \frac{b_t^M}{R_t^{gov}} + s_t = \frac{b_{t-1}^M}{\Pi_t} + m_t - \frac{m_{t-1}}{\Pi_t}$$

$$\text{Transfer from CB to Gov} \quad : \quad s_t = s + \theta_C \left(b_t^M - b^M\right)$$

$$\text{Government Budget} \quad : \quad \frac{b_{t-1}^G}{\Pi_t} + tr_t + \chi_t w_t n_t = \frac{b_t^G}{R_t^{gov}} + \tau_t w_t n_t + s_t$$

$$\text{Fiscal Rule} \quad : \quad tr_t = tr + \theta_{TR,B}\left(b_t^G - b^G\right) + \theta_{TR,Y}\left(\frac{y_t}{y} - 1\right) + \varepsilon_t$$

$$\text{Leisure/Labor} \quad : \quad \frac{Ay_t}{1-n_t} = (1-\tau_t)\, w_t$$

$$\text{Gov. Bond Interest:} \quad R_t^{gov} = \gamma_t R_t$$

$$\text{Risk Premium:} \quad \gamma_t = \gamma + \varkappa_1 \left(\frac{b_t^G}{4y} - \frac{b^G}{4y}\right)^2 + \rho_t$$

$$\text{Euler Equation Bonds} \quad : \quad \beta \frac{\sigma_t}{\sigma_{t-1}} y_t \frac{R_t^{gov}}{\Pi_{t+1}} = \frac{1-\delta}{\delta \mu_{t+1} \Pi_{t+1}} \left[b_t^H + m_t\right] + y_{t+1}$$

$$\text{Recursive Discounting} \quad : \quad \mu_t = 1 + \frac{\sigma_t}{\sigma_{t-1}} \delta \beta \mu_{t+1}$$

$$\text{Money Demand} \quad : \quad m_t = \overline{m} - \left[\frac{R_t^{gov} - 1}{R_t^{gov}}\right] \frac{1}{\nu_t y_t}$$

$$\text{Non.lin. Pricing 1} \quad : \quad F_t = \sigma_{t-1} + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{1}{\omega-1}} F_{t+1}$$

$$\text{Non.lin. Pricing 2} \quad : \quad K_t = \sigma_{t-1} \omega (1-\chi_t) w_t + \delta \xi_p \beta \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\frac{\omega}{\omega-1}} K_{t+1}$$

$$\text{Non.lin. Pricing 3} \quad : \quad \frac{K_t}{F_t} = \left[\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi}\right)^{\frac{1}{\omega-1}}}{1-\xi_p}\right]^{1-\omega}$$

$$\text{Inv. Price Dispersion} \quad : \quad \mathring{p}_t = \left[\left(1-\xi_p\right)\left(\frac{1 - \xi_p \left(\frac{\Pi_t}{\Pi}\right)^{\frac{1}{\omega-1}}}{1-\xi_p}\right)^{\omega} + \xi_p \left(\frac{\Pi}{\Pi_t} \mathring{p}_{t-1}\right)^{\frac{\omega}{1-\omega}}\right]^{\frac{1-\omega}{\omega}}$$

$$\text{Production} \quad : \quad y_t = n_t \mathring{p}_t^{\frac{\omega}{\omega-1}}$$

$$\text{Subsidy to Firms:} \quad \chi_t = 0$$

$$\text{Taylor Rule} \quad : \quad R_t = R + \phi_\pi \left(\Pi_t - \Pi\right) + \phi_y \left(\frac{y_t}{y} - 1\right) + \eta_t$$

$$\text{Profits} \quad : \quad \theta_t = y_t - (1-\chi_t) w_t n_t$$

9 Steady State

$$
\begin{aligned}
\chi &= 0 \\
\gamma &= 1 \\
\nu &= 1 \\
F &= \frac{\sigma}{1-\delta\xi_p\beta} \\
w &= \frac{1}{\omega(1-\chi)} \\
K &= \frac{\sigma\omega(1-\chi)w}{1-\delta\xi_p\beta} \\
\mathring{p} &= 1 \\
A &= (1-\tau)\,w\frac{1-n}{n} \\
y &= n \\
R^{gov} &= \frac{\Pi}{\beta} + \frac{(1-\delta)\,(1-\delta\beta)}{\delta\beta}\left[\frac{b^H}{y} + \frac{m}{y}\right] \\
R &= \frac{R^{gov}}{\gamma} \\
\overline{m} &= m + \left[\frac{R^{gov}-1}{R^{gov}}\right]\frac{1}{\nu y} \\
\frac{b^M}{y} &= \frac{b^G}{y} - \frac{b^H}{y} \\
\frac{s}{y} &= \left(\frac{1}{\Pi} - \frac{1}{R^{gov}}\right)\frac{b^M}{y} + \frac{\Pi-1}{\Pi}\frac{m}{y} \\
\frac{tr}{y} &= \frac{b^G}{R^{gov}y} + (\tau-\chi)\,w + \frac{s}{y} - \frac{1}{\Pi}\frac{b^G}{y}
\end{aligned}
$$

Note that, the higher debt held by the public, the higher is the nominal interest rate to maintain a given inflation rate.

10 Tables

Table 1: Parameters and Imposed Steady States

Parameter	Value	Description
β	0.998	Discount factor
δ	1 or 0.97	Survival probability of households
ξ_p	0.95	Calvo price stickiness
ω	1.35	Gross price markup
ϕ_π	1.5	Taylor rule coefficient on inflation
ϕ_y	0.5	Taylor rule coefficient on output
θ_C	0.01	CB to gov. transfer rule coefficient
$\theta_{TR,B}$	-0.5	Gov. to households transfer rule coef.
$\theta_{TR,Y}$	0	Gov. to households transfer rule coef.
$\varkappa_1$	0 or 0.165	Slope coefficient sovereign risk premium
$\varkappa_2$	2	Curvature coefficient sovereign risk premium
		Imposed steady states
τ	0.5	Distortionary tax rate
π	1.9	Annual inflation rate
m/y	0.25	Annual money to GDP ratio
b^G/y	0.6	Annual total gov. debt to GDP ratio
b^H/y	0.6	Annual gov. debt held by public to GDP ratio
n	1/3	Hours worked
χ	0	Subsidy to firms
γ	1	Gross sov. risk premium
ν	0.2	Level parameter utility of real money
σ/σ_{-1}	1	Discount factor shock

Table 2: Steady States and Implied Parameters

Variable	Representative infinitely lived households ($\delta = 1$)	OLG - Blanchard/Yaari households ($\delta = 0.97$)	Description
r	3.0	4.1	Nominal interest rate
$\bar{m}/y$	0.327	0.365	Satiation: annual money to GDP ratio
tr/y	0.0926	0.0906	Annual transfer to GDP ratio
w	0.74		Real wage
A	0.75		Level parameter disutiliy of labor
y	0.33		Real GDP
b^M/y	0		Annual gov. debt to GDP ratio held by CB
s/y	0.001		Annual CB transfers to gov. as ratio to GDP