Development Research Group

Study 16

DYNAMICS OF INFLATION IN INDIA: A NEURAL NETWORK APPROACH

J.C. Parikh
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Issued for Discussion

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J.C. Parikh.

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1. Introduction

The dynamics of inflation is a matter of intense debate in economics. There are several strands of thoughts explaining inflation dynamics often with reference to specific countries. The divergence in views of the different schools of thought on this issue is largely due to differences in the institutional arrangements and levels of socio-economic development of the countries, giving rise to unique structural problems. But it is important for policy makers to understand the inflation dynamics in the context of their own economies, and it is for this reason empirical research into the phenomenon of inflation processes has to be robust and continuous.

There are mainly two econometric approaches often favoured by researchers while modelling inflation dynamics of an economy. First is univariate time series modelling, where researchers assume that future price behaviour could be adequately anticipated based on past data as the data generating process of a time series is assumed to obey certain dynamic principles. As

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the univariate time series modelling is based on this set of assumptions, researchers often adopt multivariate analysis as an alternative method to find out the causal connection among the variables. Of late, within the framework of multivariate analysis, time series properties of the variables have come to be interwoven under what is called `cointegration and error correction' mechanism. In either case, the researchers impose constraints on the model about the underlying relation among the variables which may be either linear or non-linear. While this may not help fully understand the complex inflationary processes, it would still be necessary for the sake of analytical rigour to postulate the nature of the underlying relationship on the basis of definitive theoretical premises and information set.

The main purpose of this study is to understand inflation dynamics in India through the use of artificial neural network (ANN). There is hardly any work on this subject based on this method. A major advantage of this method is that non-linear behaviour of dynamic variables can be incorporated in the model. For purposes of interface, the study also provides exercises done with the help of traditional Box-Jenkins model and Vector Error Correction Model, although strict comparisons with conventional econometric techniques would be diffficult. What is important to recognise is that the ANN approach regards the process as a low dimensional non-linear deterministic system and proceeds to model the complex system that is evolving, whereas the conventional techniques assume the variables to be stochastic and apply statistical theory to model, usually in a linear frame. The variables, in the conventional approaches, are, in many cases, transformed and differenced. In the non-linear modelling using ANN, such preprocessing of the data is not done as they would remove crucial information about the evolution of the complex system that generates the observed data. It should, therefore, be borne in mind that the data being modelled itself is different under the two approaches.

The scheme of this study is as follows: A brief review of the literature on inflation is presented in Section 2. Section 3 contains details of the data used in the analysis as well as the methodology applied for modelling. Section 4 is devoted to univariate modelling using conventional as well as ANN methods. Within the framework of multivariate modelling, Section 5 compares the results of the ANN model with those under the cointegration technique. For both the univariate and the multivariate studies using ANN, the residuals (errors) have also been modelled. As a consequence, in our work we have a two step approach. In the first step the ANN models the trends and in the second step the residuals are modelled again using ANN. Section 6 studies the impulses of policy shocks on the price level. Section 7 presents concluding observations and policy implications.

2. Brief Review of Literature

The traditional explanations to the inflationary process are in terms of cost push and demand pull factors. These have been found to be inadequate to explain the complex inflationary processes. A very vast literature explaining the underlying causes and consequences of inflation under different regimes has grown since about the end of the fifties, as one can glean from such comprehensive surveys as those of Bronfenbrenner and Holzman (1963), Johnson (1963), Laidler and Parkin (1975), Hudson (1983), Gordon (1985) and McCallum (1990). Our purpose here is not to review the entire literature on inflation, but to briefly highlight some of the major ideas that would help us to understand the dynamics of inflation in India.

The two lines of thought that permeate macroeconomics relate to `activism' as against `non-activism'. Classical, including the neo-classical, and `new' classical schools, proceed on the assumption that the economy is fundamentally in equilibrium and there is, therefore, no need to adopt active policies that would interfere with the working of the market forces. Inflation, in such a framework, is the direct fall out of monetary expansion beyond what is critically needed for the economy. As against this line of thinking is the Keynesian viewpoint which questions the fundamental assumption of equilibrium of the economy and advocates intervention to pull the economy up to a higher level of output and employment. Those who espouse state activism generally agree that inflation needs to be viewed as one related to the expansion of output and employment rather than be explained purely by monetary phenomenon.

The early literature has argued that there is a positive correlation between inflation and real income or equivalently, a negative correlation between inflation and unemployment. The trade off between inflation and unemployment (the usual Phillips curve) was found to be fragile in 1970s mainly because of `stagflation'. Since there is a natural rate of growth of output and employment, the policy intervention in the long run may induce price level rather than the level of employment (Friedman, 1963, and Phelps, 1973). Moreover, if the expectation is rational, the expectation induced Phillips curve may be vertical implying the breakdown of the negative trade off between inflation and unemployment. Of late, the empirical findings suggest that there is a negative trade off between inflation and real income (Barro, 1995) which means a positive correlation between inflation and unemployment. This implies that price stability would need to be a major objective of monetary policy for sustainable growth.

There are several supply side explanations which contribute to the inflationary process in an economy. The basic supply side explanation about what causes prices to rise is the increase in the cost of production. As labour and capital are the major factors of production, the rise in real wage and real interest rate has the potential of increasing the price level. If the increase in the supply price is gradual, then it has an enduring impact on the price level which is understood to represent `core inflation'

(Eckstein, 1981). There could be sudden increase in the prices of certain factors of production (like oil prices in the 1970s) leading to increase in the general price level as the agents may not be in a position to adjust quickly in response to the shock. The impact of supply shocks on the price level is generally temporary and is expected to be dissipated over a period of time unless it is supported by monetary expansion on a continuous basis.

Another major supply side explanation of inflationary process is the downward stickiness of wages initially proposed by Keynes (1936). Prices do not adjust quickly to changes in the supply of goods because of wage rigidities and price inertia. During the seventies, however, the wage-price stickiness thesis was under severe attack from the `new' classical school of economists like Lucas (1973) and Sargent and Wallace (1975) who held that economic agents' expectations are rational and the market clearing mechanism operates in both labour and commodity markets. Therefore, wage-price stickiness cannot prevail over a medium term. In the short-run, there may be stickiness due to misperception on the part of the rational economic agents about the changes in relative prices.

Nonetheless, in recent years, there has been revival of interest in favour of wage-price stickiness spearheaded by `new Keynesians' like Ball (1991, 1995), Gordon (1985), Fisher (1994) and Taylor (1979). According to them, price rigidities may arise due to several factors like long-term wage contracting, imperfect competition, high `menu' costs and `coordination failure' among markets.

Yet another explanation of inflation dynamics, which has recently come up, is in terms of `real business cycle' theory put forward by King and Plosser (1984), Kydland and Prescott (1977) and Long and Plosser (1983). The basic idea underlying the theory is that fluctuations in prices emanate from technological shocks to the aggregate production function. When there is a positive shock to aggregate supply, there is an even bigger rise in total volume of money and credit which induces procyclical movements in money, prices and output.

In the case of open economies, the origin of inflation may be in foreign countries which may be the major trading partners of the country under consideration. If the country is under a fixed exchange rate arrangement, domestic monetary policy may have limited potency in controlling inflation. Under flexible exchange rate system, large and rapid movements in foreign exchange rate may have pervasive effects on domestic price level. Although there is no consensus as to what ultimately determines the exchange rate of a country, the proximate cause of volatility in exchange rate may be excess money supply leading to rise in price differential between the trading partners. The other major determinates of exchange rate may be the expected interest rate as well as the expected real growth rate differentials.

In the case of developing countries, however, the problem is somewhat different in the sense that there are a number of structural rigidities and bottlenecks which prevent free play of market forces. The structuralist explanation of price behaviour in the developing economies runs in terms of supply bottlenecks, rather than one of a purely monetary phenomenon.

Empirical studies of inflation in the Indian context in general tried to test the ideas of monetarists, Keynesian and structuralists (Lahiri (1981), Ramanathan and Patil (1989), Bhattacharya and Lodh (1990), Balakrishnan (1991, 1992), Ghani (1991), Ray and Kanagasabhapathy (1992)). A linear constraint is imposed in most of these studies. Besides, the open economy problem is not adequately addressed. The present study would drop the constraint of linear relation and try to capture the impact of external sector under a new method.

The brief survey of the literature made above is persuasive in the sense that we should model money, output and exchange rate to know the dynamics of inflation in an open economy.

Therefore, we develop univariate and multivariate neural network models including money, Index of Industrial Production (IIP) and foodgrain stocks (as proxy for real sector growth) and the Real Effective Exchange Rate (REER) (to represent the external sector). The dependent variable obviously is the price level/ inflation rate. For the sake of simplicity, we have presented various measures of inflation in India in Appendix I. In our modelling we use the Wholesale Price Index (WPI) and Consumer Price Index for Industrial Workers (CPI-IW) as proxies for price behaviour in India in the wholesale and retail markets, respectively.

The present study examines data from April 1975 to March 1996 covering 252 observations. The selection of the period of study is essentially needbased to have adequate data points.

3. Database and Methodology

3.1 Database

The price behaviour in India is studied in terms of the two indices, namely, All-India Wholesale Price Index (WPI) and the Consumer Price Index for Industrial Workers (CPI-IW). Although both the series generally move in tandem, several instances of divergence between them have been in evidence. The two series differ in respect of commodity coverage, number of quotations, weighting diagram, etc., and as such, do not necessarily depict an identical movement. It is, therefore, appropriate to do the empirical exercise in terms of both the series so as to avoid likely bias in choosing one against the other. In the case of multivariate analysis the exercise here has been conducted only in terms of WPI, as WPI and CPI behaved, by and large, in the same manner as in the case of univariate exercise.

There are four explanatory variables, namely, broad money, Index of Industrial Production (IIP), foodgrain stocks and real effective exchange rate in our model. Among these explanatory variables (which are used as inputs in the ANN models) broad money pertains to the last Friday of each month. For scale variable, monthly GDP would have been appropriate. As monthly GDP is not available, IIP has been used as proxy for scale variable. However, the IIP does not represent the whole economy. For the purpose of avoiding the total exclusion of agriculture, stocks of foodgrains with the public distribution system is used as a proxy for agriculture.

In a price equation, one can use interest rate as an argument. Unfortunately, no series is available on interest rate which is market determined. Only call money rate has been somewhat market related within the ceiling imposed on the banks by the Indian Banks Association. After the ceiling was withdrawn in May 1989, the call rate was found to be highly volatile. The series, thus, has a transient phase which could not be modelled in the ANN. Hence, interest rate is dropped from our analysis.

The exchange rate is considered as one of the explanatory variables in our study. It is, in fact, the external price of money which can be interpreted as a surrogate for interest rate as well. Moreover, the variable is important in influencing domestic prices through the changes in the relative prices of traded goods. Although trade-GDP ratio in India is low, it is steadily rising over time. This apart, our exports for a number of items like gems and jewellery, chemicals and engineering goods are highly import intensive. As the nominal effective exchange rate (NEER) contains the element of price effect, preference was made for trade weighted real effective exchange rate (REER) as an explanatory variable in our analysis. Nevertheless, NEER remains as the policy variable which is used to adjust REER whenever necessary. Ultimately, the authorities expect the real effective exchange rate to remain stable or corrected to the equilibrium level. In a deregulated regime, the authorities have the option to influence it through intervention.

3.2 Pre-Analysis of Time Series Data - Characterization of the Complex System

Recent developments in the study of time series data using methods of nonlinear dynamics have emphasized the importance of carrying out pre-analysis of the data. The pre-analysis characterizes the dynamic system underlying the time series data, and provides valuable information about its nature. In many cases, through the characterization, it is possible to find out whether the system is i) linear or nonlinear, ii) deterministic or stochastic, and iii) regular or chaotic. This information is very valuable for modelling the system. For this purpose various characteristics of the time series such as power spectrum, correlation dimension, embedding dimension, Lyapunov exponent, surrogate data study, determinstic vs. stochastic (DVS) plot, structure function have been investigated. The conclusions drawn from these studies are however not completely reliable because the time series is somewhat short and noisy. All the same these characteristics do provide valuable guidelines for constructing an appropriate model.

For modelling the dynamics of a complex system with variables $\vec{X}(t) = \{X_1(t), X_2(t),...X_f(t);\ t=1,...n\}$ one can, quite generally, write f first order ordinary differential equations

$$\frac{d\vec{X}(t)}{dt} = \vec{F}\,[\vec{X}(t)] \tag{3.1}$$

where $\vec{F}$ is a continuous (vector) function of all the variables X_1, $X_2,...X_f$. The functions $\vec{F}$ may also have nonlinearities. Given the initial values the Eq. (3.1) would describe the evolution of the system in the space of its variables $(X_1,...X_f)$.

In a realistic situation one does not know either the actual number of variables or the form of the function $\vec{F}$. The objective then is to use time series data of a single or a few observables to construct the dynamics of the original system. This is known in the literature [Packard *et al.* (1980), Takens (1981) and Ruelle *et al.* (1985)] as state space reconstruction of the dynamics. Implicitly it is understood that there is a relationship between the internal degrees of freedom of a deterministic system and an observable of the system in order to build a model of the measured behaviour of the system.

As long as the dynamics is not random there are two basic parameters of the time series data which one ought to know for modelling. These are the time lag τ and the embedding dimension d.

A time series $\{X(t) : t=1,...N\}$ is usually generated by measuring a dynamic variable at some frequency. It may turn out that the successive measurements $X(t)$ and $X(t+1)$ are highly correlated. This may be useful for some purposes but as we shall see not for constructing a model for a dynamical system. The present requirement (as described below) is to choose a value of time delay τ such that measurements $X(t)$ and $X(t+\tau)$ are least correlated or independent. This will ensure that only critical information in the data is employed for modelling so that the model does not contain reduntant parameters. A simple way to determine τ is to evaluate the autocorrelation function (a(T))

$$a(T) = \frac{\frac{1}{N}\sum_{m=1}^{N} (X(m+T) - \overline{X})\,(X(m) - \overline{X})}{\frac{1}{N}\sum_{m=1}^{N} (X(m) - \overline{X})^2} \tag{3.2}$$

The first minimum in the plot of a(T) vs T is usually taken to be the time lag τ. A generalization (to nonlinear domain) of the autocorrelation function is called the average mutual information. It is defined by the expression

$$I(T) = \sum_{m=1}^{N} P(X(m),\ X(m + T)) \log_2 \left[\frac{P\ (X(m),\ X(m + T))}{P(X(m))\ P(X(m + T))} \right] \quad (3.3)$$

where $P(X(m))$ is the probability of measuring $X(m)$ and $P(X(m), X(m+T)$, is the joint probability of measurements of $X(m)$ and $X(m+T)$. The distribution $P(X(m))$ is evaluated from the histogram of the frequency with which the value $X(m)$ occurs after suitably normalizing it. Similarly, for the joint distribution, one counts the frequency with which a given box is occupied in the plot of $X(m)$ versus $X(m+T)$. Again the time lag τ is chosen to be the first minimum in the plot of $I(T) \rightarrow T$. Note that if there is no clear minimum one takes τ to be that value of T where $I(T) \approx {}^1/_5 I_{max}$.

If in addition to measurement of $\{X(t)\}$ we had data on the time derivatives $\{\dot{X}(t)\}$, $\{\ddot{X}(t)\}$,.... etc. then in principle one could construct a set of differential equations that models the system. The derivatives are however not known. In a significant development, Packard *et al.* (1980) and Ruelle *et al.* (1985) proposed a method called the method of delay where a vector

$$[\vec{X}(t)] = \{X(t),\ X(t + \tau),\ldots.. \ X(t + (d - 1)\tau)\};\ t = 1,\ \ldots t_{max}] \quad (3.4)$$

when evolved in time determines the essential geometric and topological structure of the attractor of the complex system in the multidimensional space of its variables. For this purpose it is vital that the components of the delay vector $\vec{X}(t)$ be independent. This is because one can then use these components (instead of the derivatives) for modelling the system. This was the requirement mentioned above. Another crucial parameter in this geometric reconstruction of the (phase) space is the embedding dimension d. Essentially it determines the minimum number of dynamic variables needed to model the system, or the dimension of the phase space in which the attractor is embedded.

Several methods have been proposed to determine the embedding dimension d from the time series data. The method of false nearest neighbours [Abarbanel et al. (1993)] is geometric in construct. It rests on the fact that points in d dimension space may become neighbours if projected into one with a lower dimension. Thus one calculates the number of false nearest neighbours as a function of a variable dimension D. The value d at which the number of false nearest neighbour goes to zero is taken to be the embedding dimension.

It is important to realize that the determination of τ and d and their use in ANN model will enable one to follow the state space evolution (dynamics) of the time series. The parameters τ and d that are deduced from the data are therefore the most critical ones, not only for characterization but also for dynamic modelling of the system. Appendix II shows how to infer the nature of the time series using auto-correlation function, DVS plot, time delay (τ) and embedding dimension (d).

3.3 Box-Jenkins Method

A number of basic text books (Box and Jenkins (1976)) on time series analysis give the details of this model and the procedure for estimation among others and we, therefore, give here only the basic expressions.

An autoregressive moving average process of order (p,q) of a stationary Gaussian process X_t is given by

$$X_t = \sum_{i=1}^{p} \theta_i X_{t-i} + \sum_{j=0}^{q} \alpha_j v_{t-j}$$

where v_t is white noise and $\alpha_o = 1$.

The autoregressive (AR) order and the moving average (MA) order, p and q, are respectively determined depending on the partial autocorrelation and autocorrelation of the process.

3.4 Vector Error Correction Model

The Johanson's technique of Vector Autoregressive Error Correction Model (VECM) has become an important tool in time series analysis of integrated economic data. Most economic series are non-stationary and regressing such data can lead to spurious relations among the variables. Specifically, our data series are integrated of order 1, i.e. I(1), and generally any arbitrary linear combination of the series would also be I(1) giving rise to what has come to be known as spurious regression. However, there may exist certain linear combinations of the series which are I(0), in which case the relevant variables are said to be co-integrated. This notion is central to the concept of long-run relationships between variables. The co-integrating relationships are the long-run equilibrating relationships among the variables. However, in practice such equilibria are hardly ever achieved; the observed data usually deviates from the long-run equilibrium due to shocks in the variables, random effects, etc. The subsequent evolution of the variables take into account the extent of the deviation from the equilibrium relationships, and try to adjust itself towards the long-run equilibria. By explicitly introducing a term to capture the deviation of the data from the long-run relationships, the short-run dynamics of the model gets enhanced and also measures the effect of such deviations on the variables. When this specification is formulated in a Vector Auto Regressive framework we have a Vector Error Correction Model.

Formally, if X_t is an (n x 1) vector of variables which are I(1), so that ΔX is I(0), the linear system can be expressed as

$$\Delta X_t = \mu + \sum_{i=1}^{k-1} D_i \, \Delta X_{t-i} + \pi X_{t-1} + \varepsilon_t$$

where π is of rank r, $0 < r < n$. If π is correctly specified, all terms on the right hand side are I(0) and the standard distributional results apply. Once r is known, we can find $\alpha_{(n \times r)}$ and $\gamma_{(n \times r)}$ such that $\pi = \gamma\alpha'$ and $\alpha' X_{t-1}$ is I(0). The test for r is a maximum likelihood test that $\pi = \gamma\alpha'$.

The estimate of α is obtained by considering the residuals

$$R_{0t} = \Delta X_t - \sum_{i=1}^{k-1} D_i \, \Delta X_{t-i}$$

$$R_{1t} = X_{t-1} - \sum_{i=1}^{k-1} \tilde{D}_i \, \Delta X_{t-i}$$

and their second moment matrices and their cross products, S_{00}, S_{01}, S_{10}, S_{11},

$$S_{ij} = \frac{1}{T} \sum_{t=1}^{T} R_{it} R'_{jt}, \qquad i,j = 0,1$$

where the r largest eigenvalues obtained by solving the equation

$$| \lambda S_{11} - S_{10} S_{00}^{-1} S_{01} | = 0$$

$\lambda_i \geq \lambda_j \geq 0$ for $i \geq j$, determine the eigen vectors constituting the columns of $\alpha = (v_1, v_2, v_3, \ldots v_r)$. The set of eigen vectors are given by

$(\lambda_i S_{11} - S_{10} S_{00}^{-1} S_{01}) v_i = 0$, $i = 1, 2, 3, \ldots, n$.

subject to the normalization $V' S_{11} V = I$. The eigen vectors corresponding to the r largest eigen values are the cointegrating vectors. Once α is estimated, estimate of γ is given by $S_{01}\alpha (\alpha S_{11} \alpha)^{-1}$. Banerjee *et al.* (1993) is a good reference for estimation and testing of the hypotheses about the parameters involved.

3.5 Artificial Neural Network Modelling

3.5.1 Background

The task of modelling a complex system is difficult. This is due to the fact that there are many variables with unknown mutual interactions that govern the dynamics of the system. In addition, the system is quite likely to be nonlinear in nature. Traditionally modelling of such systems is carried out by first identifying crucial dynamic variables and then making assumptions about their interactions and time evolution. This enables one to set up the basic equations to study the time evolution of the system. Such an approach has obvious limitations related to the assumptions made to describe the system. For instance, it is not very clear what ought to be modified when the model is in disagreement with the data. In this context, recent developments in the application of artificial neural networks are of considerable importance.

3.5.2 Artificial Neural Network (ANN)

ANN approach has become a very valuable tool for solving certain kinds of complex problems. More precisely, ANN technology has been successfully applied to problems of i) classification of data, and ii) generalization based on features present in the data. The most important advantage of the approach is that it makes relatively weak assumptions about the dynamics of the system. It relies instead on the data and via a suitable network generates an optimal model. Another attractive feature of the ANN methodology is that it can quite easily incorporate nonlinearities.

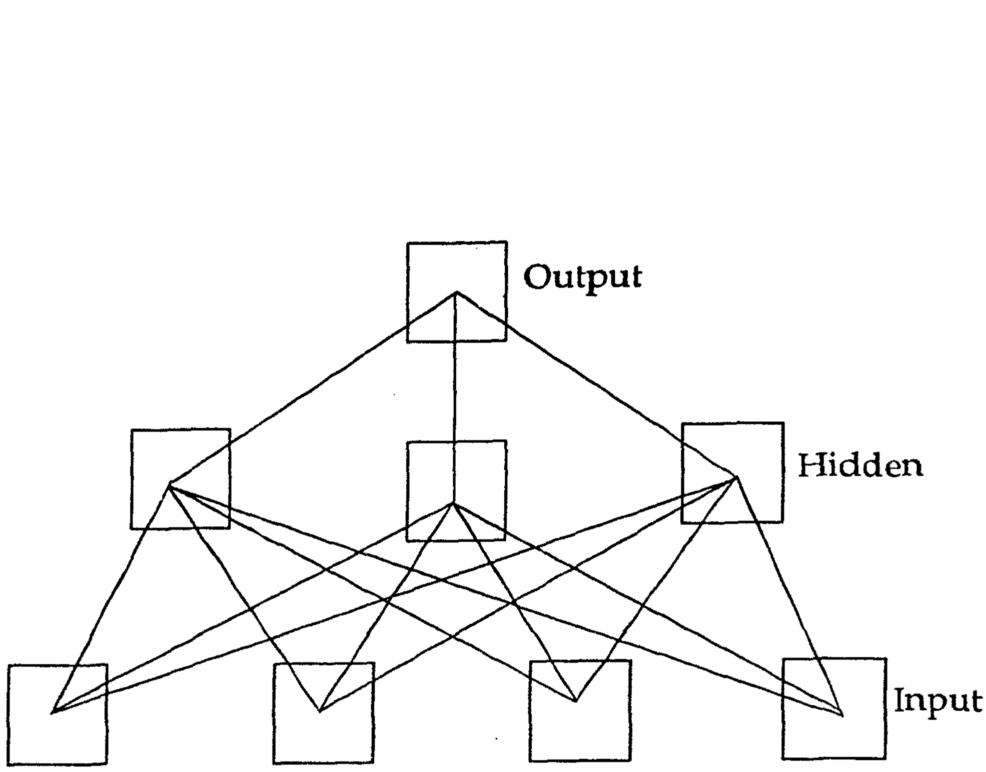


Figure 1 : A fully connected, three layered neural network with 4 neurons in the input layer, 3 in the hidden layer and 1 in the output layer.

An artificial neural network is a simplistic and idealized caricature of a human brain, which attempts to mimic some of its functions, such as associative memory, learning and adaptive information processing. It consists of a set of elements called (idealized) neurons, organized in layers with connections between the neurons. As an example, a network with 3 layers labelled input, hidden and output having 4,3 and 1 neurons respectively is shown in Fig. 1.

In this network, each neuron in the input layer is connected to every neuron in the hidden layer and similarly there is full connectivity between the neurons in the hidden and the output layers. There are no direct connections between the input and the output layers and no connections between the neurons in a layer. In the network, each connection, say between neurons labelled i and j, is assigned a weight w_{ij} which denotes the strength of the coupling between the pair (ij) of neurons. In general, one can have networks with an arbitrary number of neurons in each layer, many hidden layers and with more complex connectivity.

Within such a structural framework, the broad objective is to construct a model for the complex system. In most cases, one assumes that the model will be generated by extraction of patterns from the collected data. For this purpose, from the available data a complete set of input – output relations are extracted. The network is then trained to reproduce the input – output relations by adjusting the weights w_{ij} such that the error between the calculated network output and the desired output (data) is minimized (see Appendix III for details). The learning of the input→output relations is stored in the weights w_{ij}. Networks having different architectures thus generate different models. In principle, one therefore has an infinite number of models for a given system. In practice, however, it turns out that by increasing the number of neurons in the network or by increasing the number of hidden layers (both increase the number of parameters) the performance does not improve. One then opts for a parsimonious model where the error in the fit is small and in sample generalizations most accurate. This type of ANN model can be viewed as a nonlinear autoregressive approach without error correction. The main point, however, is that the form of nonlinearity is not assumed.

As discussed earlier, in the study of inflation using time series data of macroeconomic variables, one is able to get some guidelines about the architecture of the network from pre-analysis of the time series data. Using these guidelines ANN model has been generated which reproduces the overall trends in the data. The residuals have also been modelled using the ANN technique.

4. Modelling the Time Series - Univariate Studies

4.1 Forecast Through Conventional Box-Jenkins Method

The series M_3 was modelled after eliminating a trend equation $Log(M_3) = 9.913+0.013t$. All other series were treated on their first differences. The residuals of each series were also tested for autocorrelation and normality. The coefficients of the ARMA model and the test statistics for residual autocorrelation and normality, viz., the Box-Ljung statistic and the Jarque-Bera statistic are presented in Table 1.

Model of M_3 :

The detrended series of M_3 is modelled with an AR lag of 1 and MA lags of 12 and 24. The residuals of the ARMA model showed absence of autocorrelation and normality of distribution with Box-Ljung test statistic at 40.703 which is significant at 0.025 and the Jarque-Bera statistic at 68.993 which is significant only at 0.0 level.

Model of Wholesale Price Index :

The WPI was modelled using three Auto Regressive terms viz., lags 1, 12 and 24 and one Moving Average term of lag 3. The Box-Ljung statistic was computed at 29.139 which is significant at 0.258 showing that the residuals are not free from autocorrelation. The Jarque-Bera statistic was at 32.335 which is significant only at 0.0 indicating normality of residuals.

Model of Consumer Price Index:

An AR model with lags 1, 12 and 24 were constructed for the series CPI. Box-Ljung statistic was at 22.639 which is significant at 0.599 showing that the residuals are not free from autocorrelation. The Jarque-Bera statistic was 0.081 which is significant even at 0.96 indicating that the residuals are not normally distributed.

Model of Trade based Real Effective Exchange Rate :

The series REER was modelled as AR(1) with two interventions, viz., an additive outlier at data point 196 and innovative outliers at 204 and 252. The residuals are not free from autocorrelation as the Box-Ljung statistic at 29.088 is significant at 0.26. They are again not normally distributed as the Jarque-Bera statistic at 1.209 is significant at 0.55.

Model of Food Grain Stock :

The variable FOODS was modelled with 4 AR terms at lags 1, 3, 5, 12 and one MA term at lag 12. An additive outlier was used at point 145 and an innovative outlier at point 170. The Box-Ljung statistic (14.754 significant at 0.947) shows that the residuals are autocorrelated. However, the value of Jarque-Bera statistic 94.407 which is significant only at 0.0 shows that they are normally distributed.

The important features of Box-Jenkins models for each of the variables are summarised below. The ARMA model parameters with the standard errors are shown in Table 1 for the variables M_3, WPI, CPI, REER and FOODS.

Table 1
The coefficients and statistics of the fitted ARMA models for various variables

	M_3	WPI	CPI	REER	FOODS
AR:1 (S.E.)	-0.856023 (0.032951)	-0.391456 (0.056401)	-0.390895 (0.050988)	-0.298310 (0.058361)	-0.403408 (0.052201)
AR:3 (S.E.)					-0.137544 (0.053751)
AR:5 (S.E.)					-0.118432 (0.054414)
AR:12 (S.E.)		-0.121432 (0.053209)	-0.126304 (0.045331)		-0.208461 (0.050783)
AR:24 (S.E.)		-0.179868 (0.051447)	-0.321915 (0.046858)		
MA:3 (S.E.)		0.173711 (0.061824)			
MA:12 (S.E.)	0.391770 (0.060423)				-0.905735 (0.019498)
MA:24 (S.E.)	0.416804 (0.062303)				
MSPE:10 steps	2.04	2.89	3.87	14.37	355.82
Box-Ljung statistic (Sig. Level)	40.703 (0.025)	29.139 (0.258)	22.639 (0.599)	29.088 (0.260)	14.754 (0.947)
Jarque-Bera statistic (Sig. Level)	68.993 (0.000)	32.335 (0.000)	0.081 (0.960)	1.209 (0.546)	94.407 (0.000)

4.2 Forecast with ANN Model

As emphasized in Section 3.2, it is important to characterize the time series data before attempting to model it. Consequently, we carried out a detailed study of the characteristics (discussed earlier) of all the 7 data sets.

We first describe the general features found by us from the analysis. The autocorrelation function as well as the average mutual information for each of the time series indicate that the underlying dynamic system is essentially deterministic (non-stochastic) and correlation persists for a long time. The DVS plots imply that the dynamics has only weak nonlinearities and does not exhibit low dimension chaos. As we shall see later, we are able to construct very good linear models for M_3, WPI and CPI data sets but even the weak nonlinearities are important for modelling REER and FOODS data. The call money rate data is nearly constant for a long period (upto ~ 1989) and then shows a transient behaviour. As a result it has not been possible for us to characterize it leave alone model it. The IIP data has characteristic seasonal features which show up very clearly in the power spectrum.

Next we give in Table 2 the values of the time delay τ and the embedding dimension d obtained by using the methods described in Section 3.2 for the various data sets. The values of τ range from 3 to 10. Small values indicate that the successive points in the time series are weakly correlated whereas larger values reflect long coherence time in the data set.

Table 2
The values of time delay (τ) and embedding dimension (d) for the time series of various variables

	REER	M_3	CALL	WPI	IIP	CPI	FOODS
τ	10	10	3	6	7	6	3
d	3	3	?	3	3	2	4

The various data sets are not independent of each other and hence we determine the cross correlation, amongst them. This is shown in Table 3.

Table 3
The cross-correlation between time series of various variables at lag equal to zero

	REER	M_3	WPI	IIP	CPI	FOODS
REER	1.0					
M_3	-0.903	1.0				
WPI	-0.899	0.989	1.0			
IIP	-0.908	0.953	0.961	1.0		
CPI	-0.913	0.989	0.998	0.996	1.0	
FOODS	-0.357	0.542	0.511	0.488	0.504	1.0

The cross correlations in the data exhibit some interesting features. The most striking one is that the variable labelled FOODS is weakly correlated with all the other ones. As expected the variables CPI and WPI are highly correlated. It is interesting to note that both WPI and CPI are very strongly correlated with M_3.

We next model the time series (univariate) data sets using ANN.

Thus, for each of the data series we want to construct an input-output (I-O) model of the form

$$X_{t+1} = f\ (X_t\ X_{t-\tau}...X_{t-(d-1)\tau}) \tag{4.1}$$

Here f describes the functional relationship, τ is the time delay and d the embedding dimension for the series (see Table 2). For N data points the total number of *I-O* relations is $NV = (N-1)-(d-1)\tau$. It is worthwhile stressing that the architecture of the neural network and the total number of I-O relations directly depend on the parameters τ and d.

In order to implement a model of the form shown in Eq. (4.1) using an ANN we take d neurons in the input layer and one neuron in the output layer. The number of neurons in the hidden layer is varied between zero (linear model) to a maximum value of $\frac{N}{5(d+1)}$ where N is the total number of data points. The total number (NV) of I-O relations is divided into two sets - a training set (containing approximately 80%) of the relations and a test set having the rest of the relations. The training of the network proceeds by choosing all the weights randomly and by feeding vectors (r.h.s. of Eq. (4.1)) of the training set at the input layer. The output is calculated (see Eq. (A4) in Appendix II) and compared with the actual output in the training set data. The error is then propagated backward and the weights are corrected (see Appendix II). This procedure is repeated for the entire training set many times over until a minimum in mean square error is obtained. If the network is appropriate, a minimum of error would signify that to a large extent the I-O relations of the training set have been "learnt" by the network. The learning is stored in the weights. These weights are then used to examine the success rate in predicting the outputs for the test set. A high success rate implies that the ANN model has actually "learnt" (not "memorized") the I-O relations, and hence has the capability to generalize.

We present next the results of our ANN study of economic time series data. Our study shows that for the M_3, WPI and CPI time series a network with no hidden layer is very good. This implies that nonlinearities are not significant and linear models would work well. The optimal net for REER and FOODS data sets require a hidden layer. Thus, nonlinearities are important in these cases.

For these five time series data, we show in Figs. 2-6 the quality of the fit. Table 4 provides quantitative measures of the quality of fit. The quantity *nmse* (see Table 4) is the normalized mean square error defined by

$$nmse = \frac{\frac{1}{N} \Sigma \left(X_i(obs) - X_i(cal)\right)^2}{\text{Variance of the Observed Value}}$$

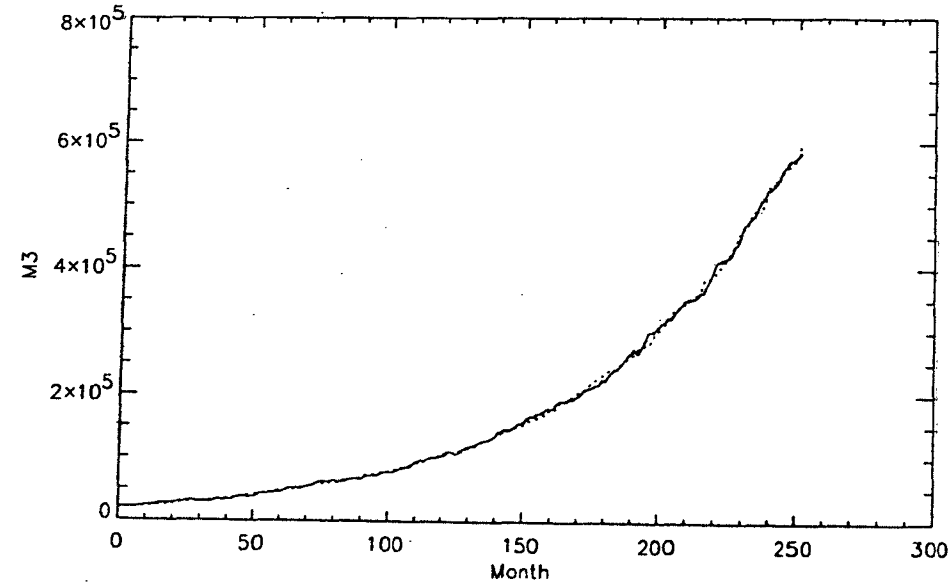


Figure 2 : The monthly observed values (dotted line) and the corresponding fitted ANN values (solid line) of the variable M_3 from April 1975 to March 1996.

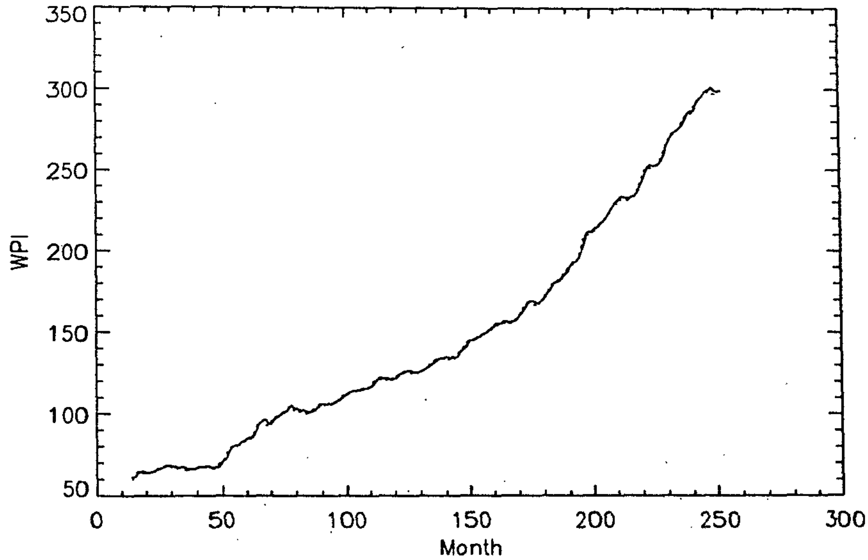


Figure 3 : The monthly observed values (dotted line) and the corresponding fitted ANN values (solid line) of the variable WPI from April 1975 to March 1996.

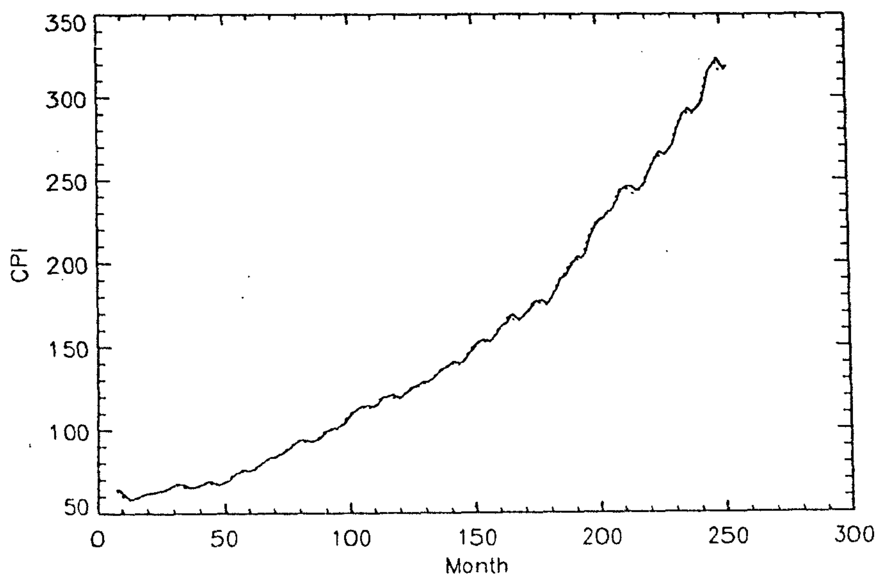


Figure 4 : The monthly observed values (dotted line) and the corresponding fitted ANN values (solid line) of the variable CPI from April 1975 to March 1996.

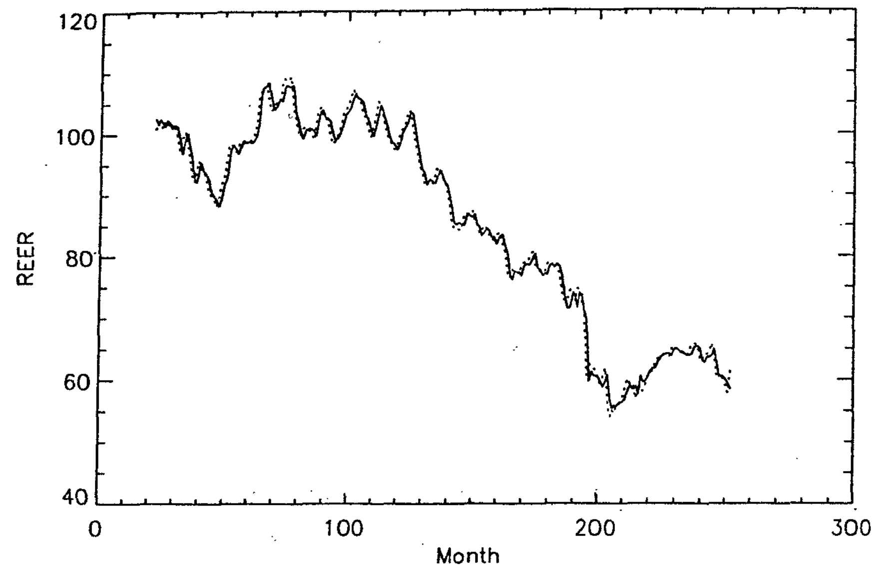


Figure 5 : The monthly observed values (dotted line) and the corresponding fitted ANN values (solid line) of the variable REER from April 1975 to March 1996.

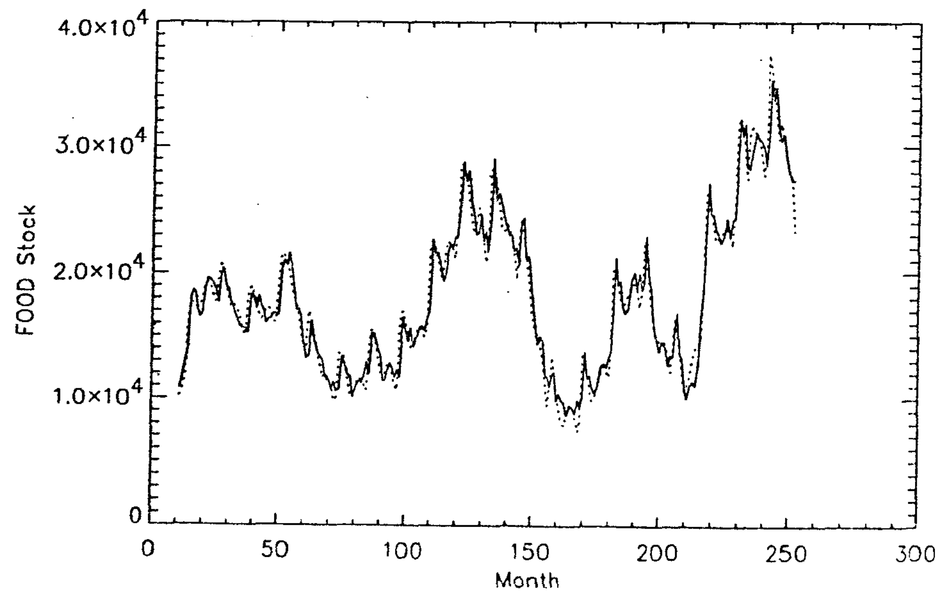


Figure 6 : The monthly observed values (dotted line) and the corresponding fitted ANN values (solid line) of the variable FOODS STOCK from April 1975 to March 1996.

Table 4
Measures of the quality of fit for the fitted and predicted values of various variables using univariate ANN models

	mean error (%) FIT	max error (%) FIT	nmse FIT	mean error % MSP (10 steps)	nmse MSP
M_3	0.76	3.72	0.0003	0.71	0.04
WPI	0.67	3.78	0.0003	0.79	0.12
CPI	0.69	3.71	0.0003	0.85	0.17
REER	1.40	15.2	0.008	1.24	1.18
FOODS	6.55	34.9	0.06171	6.46	0.46

We note that in all these cases (except FOODS data) ANN provides an excellent model so far as fitting the data is concerned. The (linear) models for M_3, WPI and CPI have superior measures compared to the nonlinear ones for REER and FOODS. Finally, due to the peculiar nature of the data on call money rate we have not been able to model it. Also, the IIP data has periodic structure and our simple network architecture and training strategy is not adequate to reproduce the periodic behaviour.

4.3 Discussion of Univariate Results

A comparison of the outcomes using the two different approaches (ARMA and ANN) to analyse the univariate time series is attempted in this section, eventhough it is known that there are several basic differences between the two methods which makes the comparison not very meaningful for the following reasons:

1. The ARMA model is applied to the detrended time series data whereas the ANN model is with respect to the actual

time series. As the pre-analysis of the time series data shows, the underlying dynamic system is essentially deterministic and hence the ANN has modelled the dominant deterministic trends of the economic time series data. This is, in fact, the first step of our univariate ANN modelling. On the other hand, if the trends in the time series data are removed, what remains can basically be characterized as "coloured noise". The ARMA then has modelled the small largely stochastic component of the dynamics. We have also verified, by carrying out the various tests for characterization of the detrended series, that they represent ``coloured noise".

2. To proceed further with a meaningful comparison of the two approaches, we have modelled the residuals left after obtaining the trends from ANN. This is the second step of our univariate ANN modelling. For this purpose, we again determined the lag (τ) and the embedding dimension (d) of the time series generated by the residuals of various variables as given in Table 5. These values are then used to construct appropriate neural networks to generate error correction.

Table 5
The values of time delay (τ) and embedding dimension (d) for the time series of various variables

	REER	M_3	WPI	CPI	FOODS
τ	3	2	3	5	3
d	3	3	3	5	4

Note that the residual time series do not show the type of variability found for the actual data. This is quite understandable.

We next compare the quality of fit, the quality of "in sample" dynamic predictions and the "out of sample" dynamic predictions. It is of value to carry out "in sample" dynamic predictions because it checks the consistency of the model and provides confidence in the model's capability to make "out of sample" predictions. It basically testifies to the robustness of the model.

The mean percentage error in fitting the data as well as the percentage error in making "in sample" 10 step dynamic predictions are shown in Table 6 for all the five series.

Table 6
Comparison of fitted and in-sample predicted values of univariate Box-Jenkins and error-corrected ANN models

	Mean Error (%) FIT		Mean Error (%) 10 step in sample dynamic prediction	
	Box-Jenkins	ANN	Box-Jenkins	ANN
M_3	0.87	0.76	2.53	0.71
WPI	0.83	0.67	1.70	0.79
CPI	0.69	0.69	1.97	0.85
REER	1.78	1.40	3.79	1.24
FOODS	4.52	6.55	18.86	6.46

We note from Table 6 that the in-sample dynamic predictions have consistently smaller errors in the ANN model. We also show in Figs. 7 to 11 the type of agreement while carrying out in sample dynamic prediction.

The single step predictions are such that they closely reproduce the fluctuations but seem to have a phase lag.

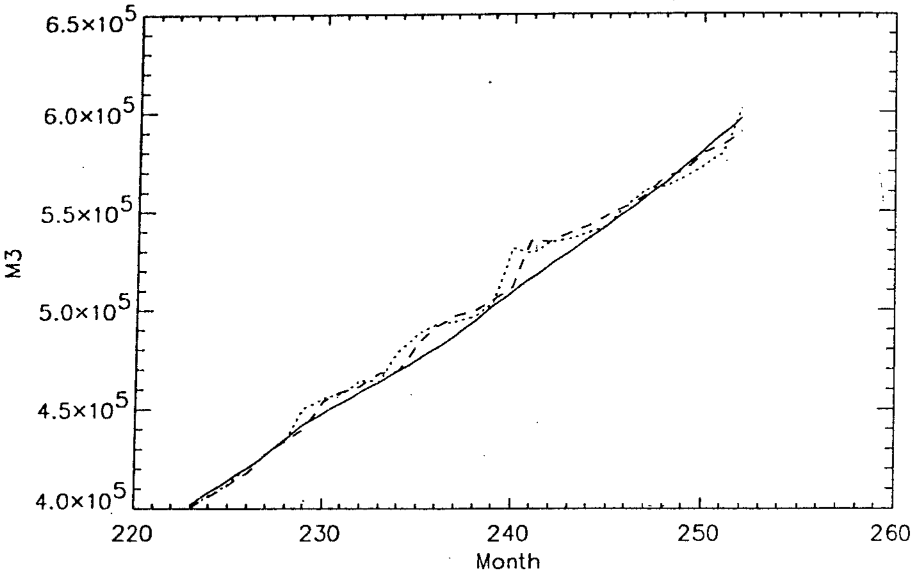


Figure 7 : A comparison of the in-sample monthly observed values (dotted line), the corresponding fitted ANN values (dashed line) and the predicted ANN values (solid line) for the variable M_3 from October 1994 to March 1996.

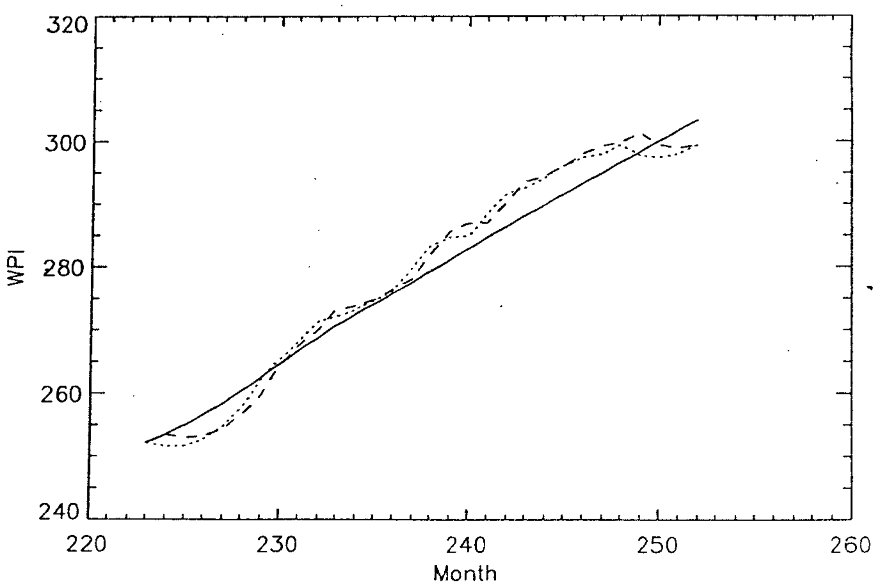


Figure 8 : A comparison of the in-sample monthly observed values (dotted line), the corresponding fitted ANN values (dashed line) and the predicted ANN values (solid line) for the variable WPI from October 1994 to March 1996.

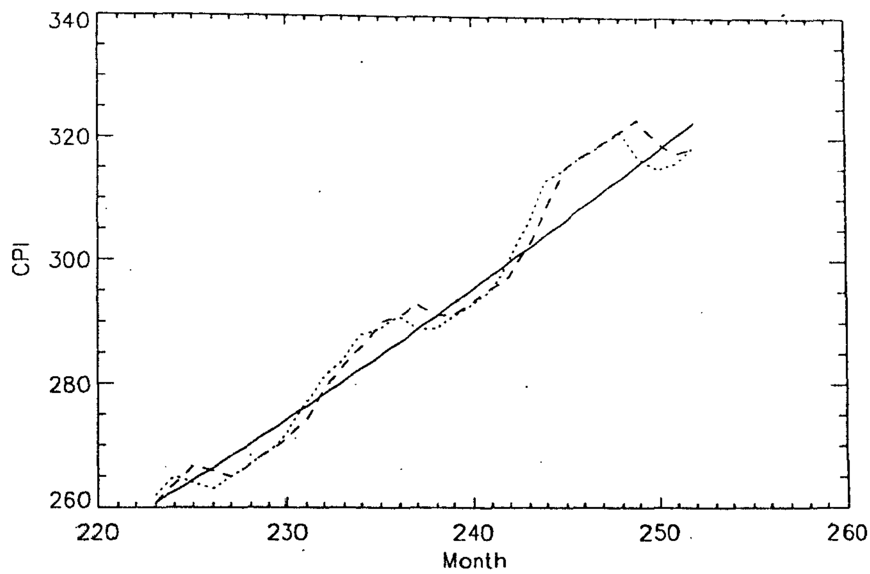


Figure 9 : A comparison of the in-sample monthly observed values (dotted line), the corresponding fitted ANN values (dashed line) and the predicted ANN values (solid line) for the variable CPI from October 1994 to March 1996.

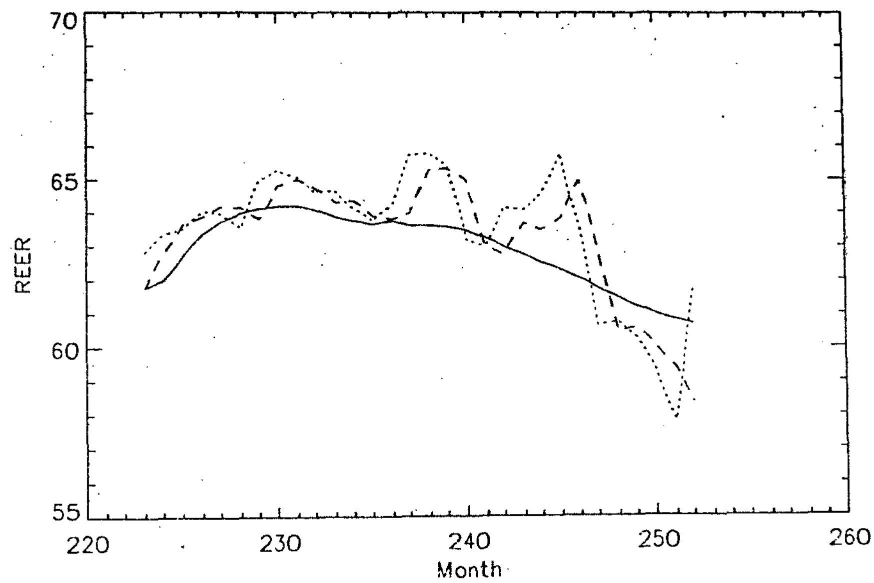


Figure 10 : A comparison of the in-sample monthly observed values (dotted line), the corresponding fitted ANN values (dashed line) and the predicted ANN values (solid line) for the variable REER from October 1994 to March 1996.

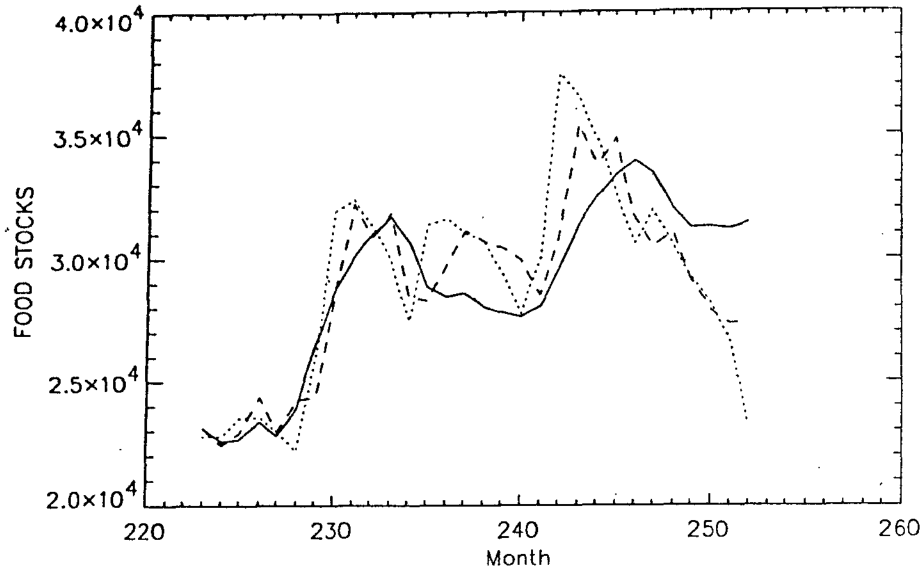


Figure 11 : A comparison of the in sample monthly observed values (dotted line), the corresponding fitted ANN values (dashed line) and the predicted ANN values (solid line) for the valuables for FOOD STOCK from October 1994 to March 1996.

The multistep predictions are also in qualitatively good agreement with the data except for the FOODS data. In the case of M_3, WPI and CPI data, the simplistic (nearly linear) predictions may be due to the linear model. It ought to be mentioned that for the REER data the B-J linear model gave erroneous trends whereas the ANN model is quite reasonable. The multistep predictions provide us with some measure of confidence concerning the error that would very likely be present in predictions made beyond the existing data.

As a real world test, forecasts have been generated for all the series from April 1996 to March 1997 using the two methods (see Table 7). The generated forecasts have been juxtaposed with actual data upto the points available for each series and forecast errors have also been presented to facilitate comparison. As can be seen from Table 7, the predictions for M_3, WPI, CPI and

REER are very similar in the two approaches. For the FOODS data the forecasts varied from 3 to 10 percent with BJ method whereas they ranged from 5 to 24 percent in the ANN approach. It may be noted that, in this case (see Table 6), the in sample dynamic predictions had much larger errors in the BJ calculations.

Finally, it is very crucial to understand that even a univariate study of time series, with the pre-analysis that we carry out and the state space approach that we implement within the ANN framework, captures the dynamics and provides valuable information about the complex system. It is also able to make good out of sample forecasts.

Table 7
Out of sample forecasts using Box-Jenkins (BJ) and ANN methods (Univariate)

obs	M_3	WPI	CPI	REER	FOOD
1996.04	612381	302.80	324	63.08	26520
BJ	605316 (1.15)	301.84 (0.32)	321. (0.76)	62.70 (0.60)	25616 (3.41)
ANN	609080 (0.54)	301.89 (0.30)	324.67 (-0.21)	62.39 (1.09)	22805 (14.01)
1996.05	618365	304.70	328	62.17	29270
BJ	615588 (0.45)	304.32 (0.12)	324.73 (1.00)	62.76 (-0.95)	31239 (-6.73)
ANN	615992 (0.38)	304.16 (0.18)	326.17 (0.56)	62.71 (-0.87)	23719 (18.96)
1996.06	624069	305.90	333	62.52	27770
BJ	621992 (0.33)	306.67 (-0.25)	329.05 (1.19)	62.61 (-0.14)	29559 (-6.44)
ANN	623241 (0.13)	306.40 (-0.16)	329.63 (1.01)	62.72 (-0.32)	24032 (13.46)
1996.07	626506	311.90	339	62.48	25740
BJ	631513 (-0.80)	309.04 (0.92)	333.57 (1.60)	62.41 (0.11)	26913 (-4.56)
ANN	633472 (-1.11)	308.31 (1.15)	333.87 (1.51)	61.73 (1.20)	23848 (7.35)
1996.08	632561	314.40	343	62.26	22060
BJ	638012 (-0.86)	310.97 (1.09)	337.12 (1.71)	62.20 (0.10)	24180 (-9.61)
ANN	647992 (-2.44)	309.95 (1.42)	337.69 (1.55)	61.14 (1.80)	22868 (-3.66)
1996.09	642256	316.80	344	62.91	19700
BJ	650719 (-1.32)	312.72 (1.29)	340.66 (0.97)	61.98 (1.48)	21666 (-9.98)
ANN	654298 (-1.97)	311.01 (1.83)	339.80 (1.22)	60.89 (3.21)	24129(-22.48)

contd..

Table 7 contd.

obs	M_3	WPI	CPI	REER	FOOD
1996.10 BJ ANN	646649 663311 (-2.58) 659868 (-2.04)	317.50 314.37 (0.99) 311.61 (1.86)	346 343.06 (0.85) 341.41 (1.33)	63.42 61.77 (2.60) 60.11 (5.22)	21140 21468 (-1.55) 28156(-33.19)
1996.11 BJ ANN	653621 669767 (-2.47) 665298 (-1.79)	319.00 316.13 (0.90) 312.43 (2.06)	349 345.38 (1.04) 343.32 (1.63)	63.00 61.55 (2.30) 59.82 (5.05)	20360 21157 (-3.91) 29763(-46.18)
1996.12 BJ ANN	658045 676392 (-2.79) 675043 (-2.58)	320.10 317.96 (0.67) 313.29 (2.13)	350 345.33 (1.33) 344.49 (1.57)	63.52 61.33 (3.45) 60.08 (5.42)	20190 20741 (-2.73) 28826(-42.77)
1997.01 BJ ANN	670646 682339 (-1.74) 686979 (-2.44)	319.80 319.94 (-0.04) 311.47 (2.60)	350 345.39 (1.32) 345.82 (1.19)	63.18 61.12 (3.26) 60.41 (4.38)	18600 19846 (-6.70) 28129(-51.23)
1997.02 BJ ANN	678359 691119 (-1.88) 696398 (-2.66)	321.40 321.68 (-0.09) 315.77 (1.75)	350 346.67 (0.95) 348.34 (0.47)	64.86 60.90 (6.11) 60.55 (6.65)	17250 18439 (-6.89) 30911(-79.19)
1997.03 BJ ANN	698056 714490 (-2.35) 705394 (-1.05)	320.70 323.22 (-0.79) 317.08 (1.13)	351 348.70 (0.66) 352.87 (-0.53)	NA 60.69 60.66	16410 16464 (-0.33) 32091(-95.56)

5. Multivariate Studies of Time Series Data

Multivariate dynamic studies are generally carried out i) to obtain functional relationships between the variables, ii) to model the dynamics and make out of sample forecasts, and iii) to examine effects of impulses given to independent variables.

5.1 Vector Error Correction Model

The Vector Error Correction Model (VECM) as described in Section 3.4 is next used to analyse the variables M_3, WPI, REER, IIP and FOODS. The variables were transformed to log form and a lag of 15 was used. Two cointegrating vectors were found among the data series at one percent level. The eigen values and the likelihood ratios are tabulated along with the hypotheses in Table 8.

Table 8
Results of co-integration analysis

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value	Hypothesized No. of CE(s)
0.151287	101.0167	68.52	76.07	None**
0.108947	62.30472	47.21	54.46	At most 1**
0.100232	35.08183	29.68	35.65	At most 2*
0.040607	10.15581	15.41	20.04	At most 3
0.001577	0.372477	3.76	6.65	At most 4

*(**) denotes rejection of the hypothesis at 5% (1%) significance level.

The two normalized co-integration equations are given in Table 9 and the corresponding residuals are plotted in figures 12 and 13.

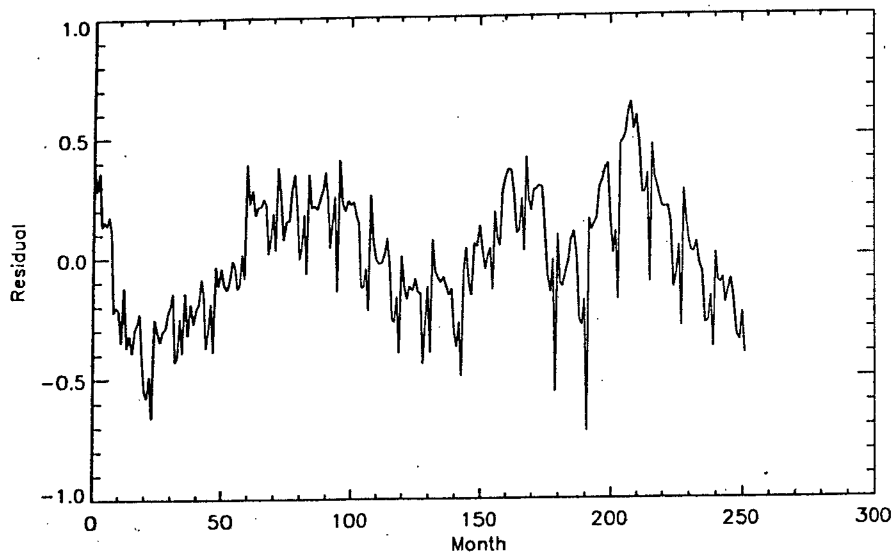


Figure 12 : Residuals of M_3 using normalized cointegrating vector #1 (see Table 9)

Table 9
Normalized coefficients of two co-integrating equations

M_3	WPI	REER	IIP	FOODS	CONSTANT
1.000000	0.000000	-0.578195	-2.585510	-0.423405	7.943789
0.000000	1.000000	0.239846	-0.851397	-0.287650	1.111365

A brief explanation would be in order in regard to the two cointegrating equations presented in Table 9.

The number of cointegration vectors in the system is determined by the rank of the coefficient matrix Π of the levels term. A likelihood ratio test is done on the eigen values of Π to determine the number of non-zero eigen values, which is the rank of Π and hence the number of cointegrating vectors.

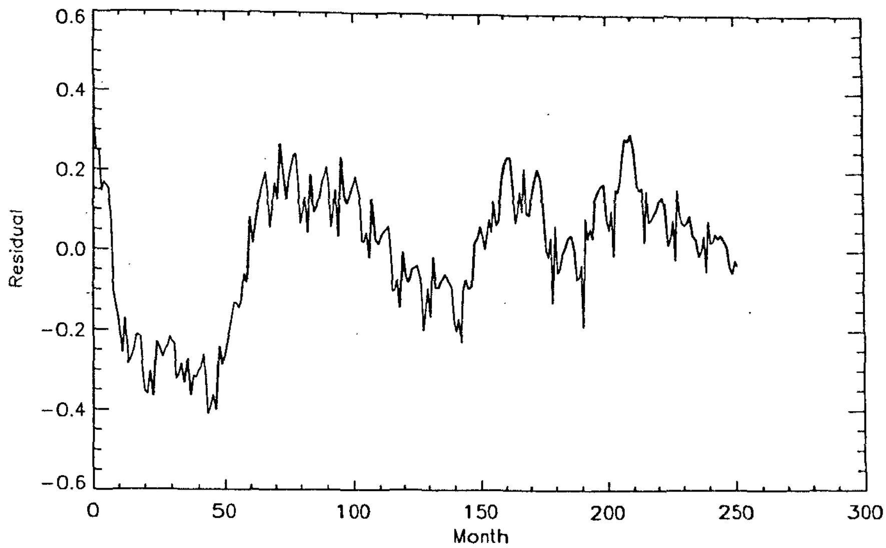


Figure 13 : Residuals of WPI using normalised cointegrating vector #2 (see Table 9).

As soon as there is more than one cointegrating vector, there is an entire subspace of vectors which satisfy the requirements of being cointegrating vectors (any linear combination of the two cointegrating vectors is itself a cointegrating vector). Consequently, any normalization is quite arbitrary. A popular approach is to normalize in a way that has the identity matrix as the coefficients on the first r variables, where r is the number of cointegrations. The original set of vectors (unnormalized) are orthogonal in the n-dimensional space, where n is the number of variables. The normalized vectors are orthogonal in the r-dimensional subspace, they are no longer orthogonal in the n-dimensional space. There is no economic meaning in the normalization we perform since any other normalization would be equally good in terms of fitting the data. It may also be noted that this method relies on the particular ordering of the variables.

As our tests indicate that there are two cointegrating vectors, we pick up the two top rows (corresponding to the two largest eigen values) of the unnormalized set of vectors. We now have a matrix B of order 2 x 5. Since presenting unnormalized vectors does not convey much meaning we have to normalize these vectors. In the case of a single cointegrating vector, we multiply the entire row with the inverse of one element so as to get unity in its place and present the results (one cointegrating equation). In the case of more than one cointegrating vector, we generalize this normalization of making one element equal unity. We express our matrix B as a partition

$$B = [B_1, B_2]$$

where B is 2 x 5, B_1 is 2 x 2 and B_2 is 2 x 3.

Now we premultiply the partitioned form by the inverse B_1^{-1} to get

$$B_1^{-1} \times B = [I, B_1^{-1} \times B_2]$$

where I is the identity matrix of order 2 x 2 and $B_1^{-1} B_2$ is 2 x 3.

The matrix $[I, B_1^{-1} \times B_2]$ is presented as normalized vectors. This way of normalization is only one mathematically convenient way of presenting the results. Owing to the use of the identity matrix I, it has the disadvantage that it makes some of the coefficients zero which makes the equations difficult to interpret economically. However, Π itself, the coefficient matrix of the levels term in the model, is invariant of any normalization that we use to report the results, as I is only one factor of Π, and the other factors say A is postmultiplied by B_1 so that for $\Pi = A \times B$, and for any nonsingular B_1,

$$\begin{aligned}(A \times B_1) \times [I, B_1^{-1} \times B_2] &= (A \times B_1) \times (B_1^{-1}) \\ &= A \times (B_1 \times B_1^{-1}) \\ &= A \times I \times B \\ &= A \times B = \Pi\end{aligned}$$

An Error Correction Model with variables transformed to log form was estimated for ΔM_3, ΔWPI, ΔREER, ΔIIP and ΔFOODS on their own previous 15 lags and on the cointegrating equations shown in Table 9. The quality of fit is shown in Table 10.

Table 10
The quality of fit obtained for various variables using Vector Error Correction Model

Residuals	M_3	WPI	REER	IIP	FOODS
Mean Abs. % Error	0.5237	0.4312	1.1420	1.8990	2.8926
Max. Abs. % Error	2.9768	2.3500	10.8180	8.3911	15.6303
NMSE	0.00013	0.00013	0.00544	0.00485	0.01276

5.2 ANN Model

A number of multivariate ANN studies were carried out by us with different objectives in view. It ought to be mentioned that to the best of our knowledge these are the first such studies using the ANN method. A part of the work was concerned with exploring functional relationships between the five variables M_3, WPI, REER, IIP and FOODS. More precisely, we use the ANN approach to examine nonlinear relations amongst these macroeconomic variables. Thus, we consider (in turn) one of the five variables as a dependent variable and express it in terms of the other variables. In another investigation we wanted to examine the simultaneous time evolution of the system allowing all the five [WPI, M_3, REER, IIP, FOODS] dynamic variables to influence each other. It should be mentioned that the amount of data is not enough for a full vector to vector ANN mapping. In view of this we have carried out vector $\rightarrow$ scalar multivariate ANN models. Further, we also estimated the effect of giving a $\pm$ 5% impulse in M_3 and $\pm$2.5% impulse REER on WPI.

We first discuss the nonlinear relations amongst the variables. For this purpose we therefore construct a sequence of ANN models given below.

$$(M_3)_t - f_1(REER, WPI, IIP, FOODS)_t = 0 \tag{5.1}$$

$$(WPI)_t - f_2(REER; M_3, IIP, FOODS)_t = 0 \tag{5.2}$$

$$(REER)_t - f_3(M_3, WPI, IIP, FOODS)_t = 0 \tag{5.3}$$

$$(IIP)_t - f_4(REER, M_3, WPI\ FOODS) = 0 \tag{5.4}$$

$$(FOODS)_t - f_5(REER\ M_3, WPI, IIP) = 0 \tag{5.5}$$

We found that in all the cases it was necessary to include a hidden layer in the optimal net i.e., the nonlinearaties are important. The quality of the fit is shown in Table 11. The moments of the residuals are shown in Table 12. We note that the models of Eq. (5.1), (5.2) and (5.3) give better fit compared to the others as they have *nmse* below 2%. However, we would like to concentrate on two most important economic variables, such as M_3 and WPI, which are the basic focus of our analysis. It ought to be stressed that although we have obtained these nonlinear relations it is not possible to specify how many of them are independent.

Table 11
The quality of fit of nonlinear relations for various variables using ANN

	Max % Error	Mean % Error	RMS Error	NMSE
M_3	11.35	2.60	4525.25	0.00084
WPI	4.73	1.36	2.23	0.001
REER	11.54	2.03	2.16	0.0165
IIP	22.21	4.49	10.84	0.0317
FOODS	103.13	14.71	2649.73	0.1628

Table 12
A comparison of the lowest two moments of residuals obtained from ANN

	Nonlinear Relations using ANN	
	M3	WPI
Mean	44.21	0.0068
Std	4524.02	2.23

We compare the nonlinear relations amongst the variables with the data in Figs. 14-15.

Next we carry out full dynamic multivariate calculations incorporating the lag structures and the embedding dimension of the variables in the neural network framework. At the conceptual level this is an improvement over the VECM. This is because the level terms are modelled dynamically and further ANN can tackle nonlinearities in the system.

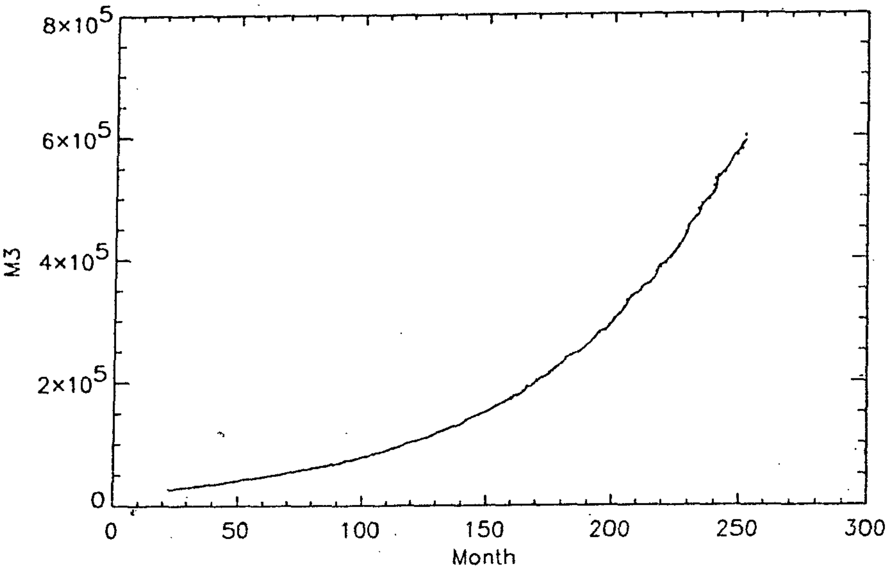


Figure 14 : Nonlinear relation for M_3 obtained with ANN (see (eq. 5.1)). Observed values (dotted line) and ANN values (solid line).

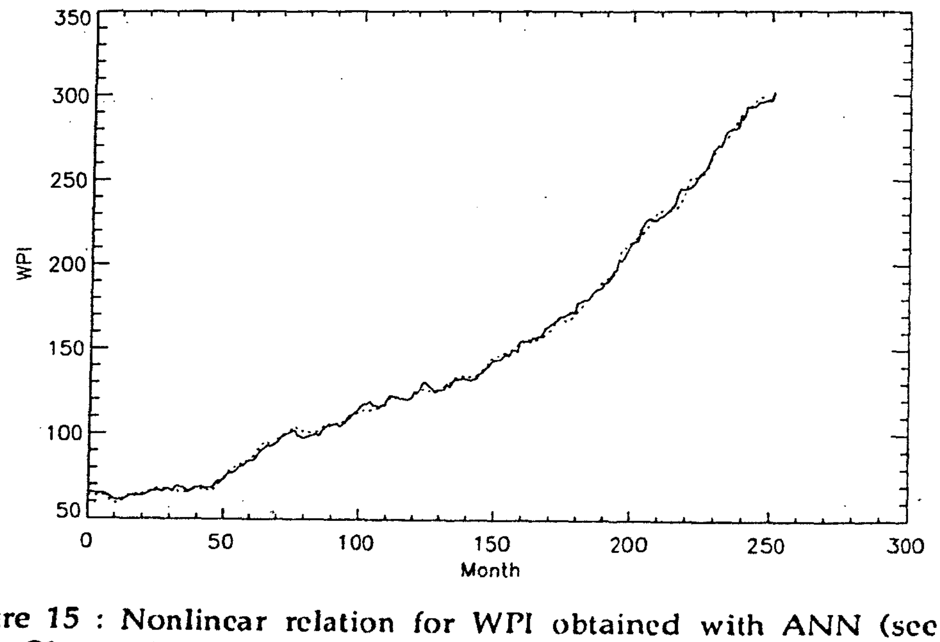


Figure 15 : Nonlinear relation for WPI obtained with ANN (see (eq. 5.2)). Observed values (dotted line) and ANN values (solid line).

We therefore developed multivariate vector→scalar ANN models to study the time evolution of all the five variables. It should be pointed out that a complete multivariate vector→vector model could not be generated due to lack of data points. Let us define a five component vector

$$\vec{X} \equiv (X_1, X_2, X_3, X_4, X_5) \equiv (M_3, WPI, REER, IIP, FOODS)$$

The multivariate vector→scalar models that we have constructed are of the type

$$X_{i,t+1} = f_i(X_{1,t}\ X_{1,t-\tau_1} \cdots X_{1,t-(d1-1)\tau_1} \cdots X_{5,t}\ X_{5,t-\tau_5} \cdots X_{5,t-(d5-1)\tau_5}) \tag{5.6}$$

for each variable X_i (i=1,...5). In eq. (5.6), (τ_i, d_i) refer to the lag and the embedding dimension respectively for the variable X_i. These are given in Table 2. Optimal neural networks were obtained in each case and the quantitative aspects of the fit are shown in Table 13.

Table 13
Quality of fit of multivariate vector-scalar ANN models

Model	Quality Measures	Data Sets				
		REER	M_3	WPI	IIP	FOODS
ANN	Mean Abs. Error (%)	1.34	0.82	0.55	4.14	5.99
	Max. Abs. Error (%)	17.30	3.91	2.78	21.59	33.16
	NMSE	0.0078	0.00028	0.00025	0.036	0.042
ANN with Error Correction	Mean Abs. Error (%)	1.10	0.67	0.47	2.30	3.39
	Max. Abs. Error (%)	8.13	3.83	2.42	8.80	21.25
	NMSE	0.0046	0.00012	0.00016	0.0070	0.012

The results are as expected. The dynamics of trends in the data is modelled quite well by the data and error correction improves the fit. The next step is to make forecasts.

5.3 Forecasts in the Multivariate Model

In order to make out of sample forecasts, we again follow a two step approach. In the first step, we determine the trends by constructing models according to Eq. (5.6). The second step consists of using ANN to model the residuals. Clearly, in the multivariate case we need to incorporate cross correlations amongst the variables. We have therefore determined the partial auto correlation and cross correlation with a whole range of lags. The statistically significant lags are then included in the determination of the input-output relations. These are then modelled by ANN in a dynamic system to make it comparable with the error correction time series model. In fact, our dynamic model is a nonlinear ANN implementation of the linear transfer function model involving cross correlations as discussed by Box and Jenkins ((1976). The explicit equations for the residuals δX_1 is

$$
\begin{aligned}
(\delta X_1)_{t+1} = f_1\big((\delta X_1)_t, (\delta X_1)_{t-\tau 1} \ldots (\delta X_1)_{t-\tau\alpha 1};\ &(\delta X_2)_t, (\delta X_2)_{t-\tau 2} \ldots (\delta X_2)_{t-\tau\alpha 2} \\
&\vdots \\
&(\delta X_5)_t, (\delta X_5)_{t-\tau 5} \ldots (\delta X_5)_{t-\tau\alpha 5}\big)
\end{aligned}
\tag{5.7}
$$

Here $\tau_1,\ldots\tau_{\alpha 1}$ are significant autocorrelation terms and $\tau_i\ldots\tau_{\alpha i}$ are the significant cross correlation terms between the variable 1 and i. There are similar equations for the residuals $\delta X_2,\ldots\delta X_5$. The various τ's are shown in Table 14. The quality of fit to the observed values improves after applying the error correction procedure (see Table 13). The dynamic model was then used to forecast data from April 1996 to March 1997 as seen in Table 15. This horizon falls entirely outside the sample data. As in the univariate analysis, the actual data as well as the forecast errors are presented along with the forecasts for easy comparison.

Table 14
Lags in partial autocorrelations and cross-correlations used for modelling residuals

	M_3	WPI	REER	IIP	FOODS STOCKS
M_3	6,11,16,23	11	18	10,11,35	3,4,21,27,33
WPI	—	1,2,4,19,22	—	5,7,10,17, 19,24	12,16,20,22
REER	—	1	1,7,20	4,8,10,16	1
IIP	18	19,24,29, 31,36	8	11,12,13,14, 24	2,9,10,12,22, 24,34,36
FOODS STOCKS	12,17,18, 32	1,4,14,26, 38	—	2,3,12,14,24, 25,30,36,38	1,2,4,5,10,12, 13,24,25

We note that for M_3, WPI and IIP the predictions with ANN are slightly better than, and in any case, comparable to those of VECM. For REER the ANN predictions are consistently better compared to VECM. As regards the FOODS stock data, ANN gives predictions with large errors; this is because, in our view, for the multivariate studies using ANN, the data are still not enough. We believe that predictions would be better with more data.

Table 15
Multivariate out of sample forecasts

obs	M_3	WPI	REER	IIP	FOODS
1996.04	612381	302.80	63.08	278.40	26520
VECM	606177 (+1.02)	299.49 (+1.52)	61.80 (+1.85)	285.29 (-2.44)	23144.12 (+12.71)
ANN	613964 (-0.26)	301.83 (0.32)	62.08 (1.61)	283.81 (-1.91)	25099 (5.66)
1996.05	618365	304.70	62.17	289.20	29270
VECM	612024 (+1.03)	299.28 (+1.94)	61.05 (+1.63)	292.80 (-1.25)	26984.06 (+7.80)
ANN	617813 (0.09)	301.63 (1.02)	61.62 (0.89)	292.03 (-0.97)	30042 (-2.57)
1996.06	624069	305.90	62.52	279.40	27770
VECM	612879 (+1.66)	303.33 (+1.00)	59.26 (+5.06)	285.71 (-2.18)	26138.75 (+5.87)
ANN	625584 (-0.24)	301.25 (1.54)	60.74 (2.93)	304.79 (-8.33)	24681 (12.53)
1996.07	626506	311.90	62.48	285.70	25740
VECM	617737 (+1.25)	306.17 (+2.27)	58.16 (+6.96)	293.52 (-2.96)	23699.89 (+7.92)
ANN	625990 (0.08)	301.99 (3.28)	61.30 (1.92)	307.89 (-7.21)	24771 (3.91)
1996.08	632561	314.40	62.21	289.00	22060
VECM	626343 (+0.70)	307.10 (+2.23)	57.27 (+7.45)	301.15 (-4.20)	21722.73 (+1.53)
ANN	655282 (-3.47)	304.07 (3.40)	60.48 (2.86)	312.60 (-7.55)	27558 (-19.95)
1996.09	642256	316.80	62.91	303.40	19700
VECM	638471 (+0.15)	308.44 (+2.67)	56.54 (+10.13)	297.64 (+1.90)	18926.04 (+3.93)
ANN	657736 (2.35)	303.04 (4.54)	61.43 (2.41)	292.41 (3.76)	26793 (-26.47)
1996.10	646649	317.50	63.43	301.10	21140
VECM	648723 (-0.76)	307.70 (+3.09)	55.22 (+12.94)	292.99 (+2.69)	18221.96 (+13.80)
ANN	655147 (-1.30)	303.87 (4.49)	61.45 (3.22)	302.59 (-0.49)	27698 (-23.68)
1996.11	653621	319.00	63.00	296.20	20360
VECM	656364 (-0.42)	304.59 (+4.52)	54.41 (+13.63)	318.70 (-7.60)	17029.38 (+16.36)
ANN	656597 (-0.45)	305.44 (4.44)	62.22 (1.25)	301.10 (-1.63)	24870 (-18.13)
1996.12	658045	320.10	63.52	318.80	20190
VECM	663809 (-0.86)	302.37 (+5.54)	53.75 (+15.38)	346.54 (-8.70)	16402.25 (+18.76)
ANN	665992 (-1.19)	306.14 (4.56)	61.50 (3.28)	311.67 (2.29)	22333 (-9.60)
1997.01	670646	319.80	63.18	315.60	18600
VECM	671220 (-0.09)	301.94 (+5.58)	53.01 (+16.10)	348.83 (-10.53)	16828.83 (+9.52)
ANN	670393 (0.04)	307.69 (3.94)	60.64 (4.19)	298.54 (5.71)	24118 (-22.88)
1997.02	678359	321.40	64.86	312.80	17250
VECM	682456 (-1.08)	299.75 (+6.74)	52.13 (+21.48)	337.36 (-7.85)	16698.47 (+3.20)
ANN	704848 (-3.76)	306.22 (4.96)	61.25 (5.89)	300.11 (4.23)	24312 (-29.05)
1997.03	698056	320.70	NA	NA	16410
VECM	701168 (-0.45)	298.49 (+6.93)	50.93	378.57	15393.34 (+6.20)
ANN	693521 (0.65)	305.59 (4.94)	60.51	311.17	23327 (-29.65)

6. Impulses and Policy Implications

In our scheme of analysis, IIP and foodgrain stocks are outside the control of the monetary authorities. The remaining explanatory variables are broad money and real effective exchange rate which may be treated as policy variables. Whether the authorities have full control over them is an issue which is not the focus of discussion here. Again adjusting money supply and/or real effective exchange rate depends on the perception about the level of equilibrium. In order to arrive at an equilibrium on an *ex ante* basis, the policy makers should have a fairly accurate idea about the demand for domestic money as well as foreign exchange. The latter is considered as one of the major determinants of the exchange rate. The factors influencing the demand for money and exchange rate are beyond the scope of this study.

A caveat is probably necessary as regards equilibrium. In *ex post* sense, the market is always in equilibrium. Theoretically, equilibrium could be obtained by changes in several equilibrating variables which may be price level or interest rate or exchange rate or such other variables depending on the behavioural situations. It would, however, be ideal if equilibrium is consistent with the broad policy objectives set out on an *a priori* basis.

Keeping in view the objective of price stability as the core of monetary policy, we simulated our ANN model by giving four types of shocks at two different points of time and studied their impact on the price level. First, shock is given in terms of broad money and its impulse on prices is studied. Secondly, the impulse on prices is studied by giving shock to REER only. Thirdly, a combination of shocks, namely, an increase in money supply and a rise in REER is visualised and the resultant impulses on prices are examined. Fourthly, the impact of a combined shock of increase in M_3 and fall in REER is also studied.

The third and fourth types of shocks need some elaboration. *Ceteris paribus,* an increase in M_3 tends to raise domestic prices. Unless the prices of major trading partners rise in proportion, the price differential between domestic and overseas markets would tend to widen. In such a situation, if the nominal exchange rate is not adjusted downward in proportion to the price differential, the real effective exchange rate would tend to appreciate. When capital inflow is more than the current account deficit, the market forces would improve the supply of foreign exchange and thereby prevent depreciation of domestic currency. Hence, a combination of rise in M_3 and appreciation of REER would be a realistic situation whose impact needs to be studied. However, an increase in M_3 could also be combined with a fall in REER, on the following rationale. Consider a situation where there is no capital inflow in excess of external current account deficit. Assume also a relative increase in money supply. The price differential then may widen, bringing about a concurrent downward adjustment of NEER. If a free fall of the NEER is allowed, there could be over correction leading to a fall in REER.

The two points of impulse in April 1993 and April 1994 have been chosen essentially for purposes of convenience. The first point of shock leaves 35 observations ahead during which the impact of production cycles on price behaviour could be tracked in addition to the monetary shock. The second shock leaves 23 observations ahead, which is fairly long to observe the short run impact of policy changes on the price level.

Thus, the impact of the following policy shocks given in April 1993 and in April 1994 are examined:

(i) ±5 per cent shock given to M_3;

(ii) ±2.5 per cent shock given to REER;

(iii) +5 per cent shock to M_3 combined with +2.5 per cent shock given to REER, and

(iv) + 5 per cent shock to M_3 combined with -2.5 per cent shock given to REER.

The impact of ±5% change in money supply shock on WPI, given in April 1993, is plotted in Fig. 16. The solid line is the estimated model, while the lines represented by dash and dot (-.-) and dash (-) are simulated paths of price corresponding to +5% and -5% shocks in the money supply, respectively.

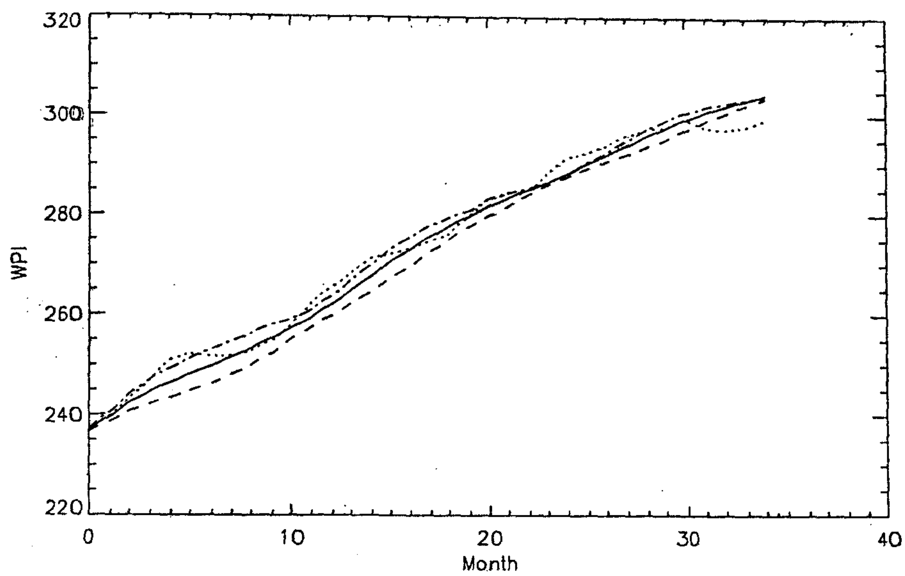


Figure 16 : The monthly values of variable WPI before and after the impulse is given in April 1993. The values range from May 1993 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the impulse of -5% in M_3 (dashed line), (iv) Predicted values after the impulse of + 5% in M_3 (dash-dot line).

The immediate impact of rise/fall in money supply on the price level is rather modest and has a tendency to return close to the model after a period of about 10 months. Thereafter, the oscillation in prices remains subdued and reverts to the model at about the 24th month. The oscillation of prices that begins once again beyond the 24th month is somewhat of a lower magnitude which is gradually dampened. Subsequently, the simulated prices move in tandem with the model. Barring this oscillation which needs explanation, it is established that increase in money supply is associated with rise in WPI and *vice versa. Prima facie,* following the monetary shock, the oscillation of prices upto 10 months looks generally pronounced which, however, tapered off gradually. Statistically, we can recall from the pre-analysis of the data discussed earlier, that the series on M_3 was least correlated with its 10th lag (τ=10). The modest rise in prices immediately after the shock could be explained in terms of sudden rise in purchasing power in the hands of the people who do not necessarily spend the entire rise in money income. The notion of `buffer stock' holding of money may fit into this situation as the likely explanation. Unless higher order of money supply is sustained, the inflation rate has a tendency to peter out over a period of 2 years. There is high degree of symmetry in the oscillation of predicted values around the model which probably offers the band within which actual price level is expected to fluctuate. However, the observed series of WPI seems to have been outside the band on certain occasions implying the likely impact of non-monetary factors influencing the price level.

The predicted values of WPI before impulse and after impulse are shown in Table 16. When a -5% shock was given to M_3, the predicted prices remained below the model values throughout the sample period. The cycle seems to have been weakened after 24 months. In the case of +5% shock to M_3, prices are higher than the observed values on 32 occasions out of 35 months. The percentage variations over the actual were, however, less severe than the earlier case of negative shock. A few instances of actual WPI remaining above values corresponding to the positive shock implies the influence of non-monetary factors on the prices as mentioned earlier.

Table 16
Impulse response on WPI (±5% monetary shock given in April 1993)

Month/Year	Model WPI before impulse	Predicted WPI after -5% M_3 shock	% variation	Predicted WPI after +5% M_3 shock	%variation
May, 93	236.90	236.61	-0.12	237.19	0.12
June, 93	239.58	238.66	-0.38	240.48	0.38
July, 93	242.31	240.60	-0.71	243.99	0.69
August, 93	244.66	242.16	-1.02	247.09	0.99
September, 93	246.35	243.38	-1.21	249.22	1.17
October, 93	248.11	244.82	-1.33	251.25	1.27
November, 93	249.68	246.14	-1.42	253.02	1.34
December, 93	251.39	247.91	-1.38	254.58	1.27
January, 94	253.21	249.89	-1.31	256.19	1.18
February, 94	255.28	252.18	-1.21	258.03	1.08
March, 94	257.16	255.02	-0.83	258.93	0.69
April, 94	259.45	257.51	-0.75	260.97	0.59
May, 94	261.93	259.76	-0.83	263.59	0.63
June, 94	264.74	262.09	-1.00	266.76	0.76
July, 94	267.64	264.58	-1.14	269.94	0.86
August, 94	270.55	267.23	-1.23	273.01	0.91
September, 94	273.12	269.70	-1.25	275.60	0.91
October, 94	275.58	272.46	-1.13	277.75	0.79
November, 94	277.75	275.12	-0.95	279.46	0.62
December, 94	279.74	277.74	-0.71	280.92	0.42
January, 95	281.84	279.72	-0.75	283.21	0.49
February, 95	283.56	281.98	-0.56	284.46	0.32
March, 95	285.11	284.15	-0.34	285.45	0.12
April, 95	286.66	286.13	-0.18	286.59	-0.02
May, 95	288.45	287.84	-0.21	288.44	-0.00
June, 95	290.32	289.37	-0.33	290.58	0.09
July, 95	292.21	290.94	-0.43	292.74	0.18
August, 95	294.11	292.27	-0.63	295.15	0.35
September, 95	295.96	293.78	-0.74	297.31	0.46
October, 95	297.78	295.41	-0.80	299.31	0.51
November, 95	299.44	297.25	-0.73	300.79	0.45
December, 95	300.89	299.06	-0.61	301.92	0.34
January, 96	302.15	300.74	-0.47	302.79	0.21
February, 96	303.25	302.31	-0.31	303.47	0.07
March, 96	304.31	303.70	-0.20	304.21	-0.03

The money shocks given in April 1994 depict, more or less, similar oscillations (Fig. 17). A cycle of a little over 10 months in the price behaviour is observed in this case (Table 17). There are, however, a few noticeable differences between these two shocks. In the first case, the immediate impact of the monetary shock given in April 1994 was rather more pronounced compared to the same given in April 1993. As the money supply was already high due to capital inflows, adjustment on either side indicates noticeable difference in the prices. However, the prices converged to the model after a period of about 10 months and thereafter the next oscillation started. Secondly, the oscillation in prices in the second spell was relatively more modest compared to the previous shock. Thirdly, the actual price level which was occasionally outside the lower band in the first spell of oscillation, moved within the band in the second spell. Fourthly, the second convergence seems to have been achieved at the 19th month, which roughly coincides with the 10-month cycle, at which the impact of shock might have petered out.

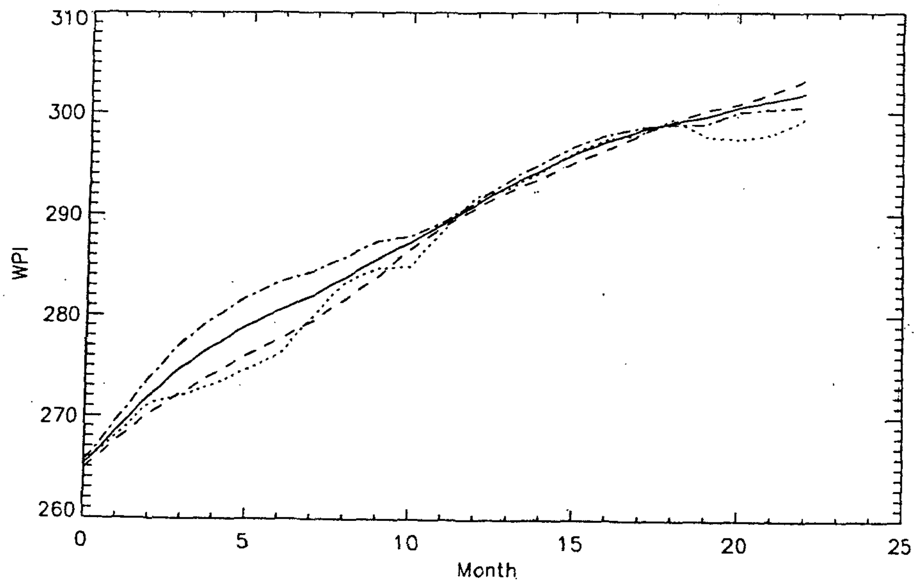


Figure 17 : The monthly values of variable WPI before and after the impulse is given in April 1994. The values range from May 1994 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the impulse of -5% in M_3 (dashed line), (iv) Predicted values after the impulse of + 5% in M_3 (dash-dot line).

Table 17
Impulse response on WPI (±5% monetary shock given in April 1994)

Month/Year	Model WPI before impulse	Predicted WPI after -5% M_3 shock	% variation	Predicted WPI after +5% M_3 shock	%variation
May, 94	265.21	264.87	-0.13	265.55	0.13
June, 94	268.66	267.66	-0.37	269.65	0.37
July, 94	272.06	270.27	-0.66	273.85	0.66
August, 94	274.90	272.39	-0.91	277.38	0.90
September, 94	277.06	274.24	-1.02	279.85	1.01
October, 94	279.01	276.12	-1.04	281.85	1.02
November, 94	280.54	277.68	-1.02	283.34	1.00
December, 94	281.97	279.47	-0.89	284.38	0.85
January, 95	283.76	281.62	-0.75	285.81	0.72
February, 95	285.67	283.86	-0.63	287.41	0.61
March, 95	287.27	286.61	-0.23	287.87	0.21
April, 95	289.07	288.79	-0.10	289.29	0.08
May, 95	290.95	290.63	-0.11	291.18	0.08
June, 95	292.87	292.29	-0.20	293.35	0.16
July, 95	294.37	293.60	-0.26	295.04	0.23
August, 95	295.96	295.11	-0.29	296.69	0.25
September, 95	297.17	296.39	-0.26	297.83	0.22
October, 95	298.31	297.90	-0.14	298.59	0.09
November, 95	299.04	299.10	0.02	298.85	-0.06
December, 95	299.64	300.27	0.21	298.90	-0.25
January, 96	300.55	300.89	0.11	300.10	-0.15
February, 96	301.21	301.98	0.26	300.39	-0.27
March, 96	301.93	303.25	0.44	300.67	-0.42

The second set of independent shocks to the extent of ±2.5 per cent is given to REER. The outcomes of this shock, given in April 1993, are plotted in Fig. 18 which indicate that the immediate impact of this shock on prices is negligible. Given the low degree of openness of the economy, changes in REER may not immediately affect the Wholesale Price Index to a noticeable extent. However, the price level is noticeably affected from the 10th month and the impact is gradually accelerated thereafter (Table 18). A downward adjustment of REER, which is interpreted as implied devaluation, ultimately turns out to be inflationary and *vice versa.* Changes in REER seems to have affected the price level after a time lag. Since depreciation and/or appreciation of REER affect the cost structure of the economy, the impact of the shock persisists.

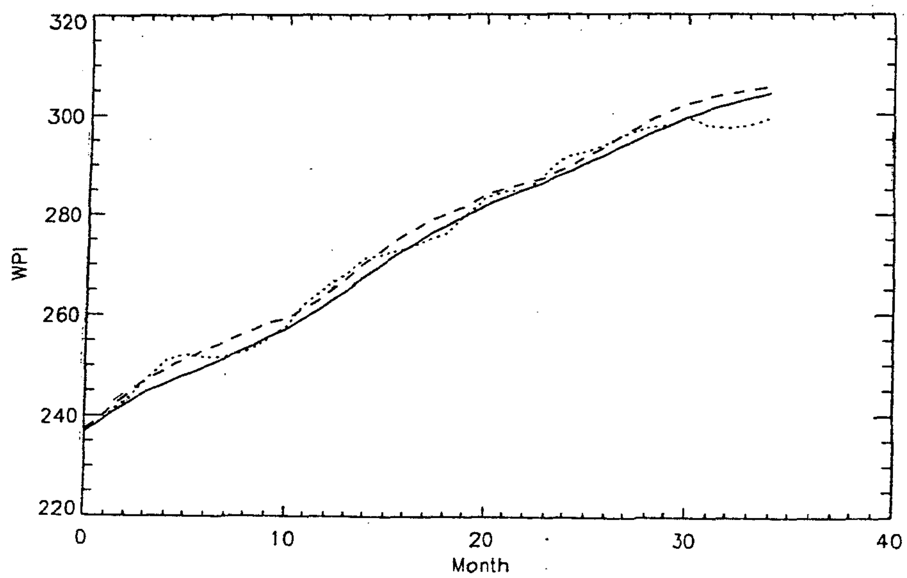


Figure 18 :The monthly values of variable WPI before and after the impulse is given in April 1993. The values range from May 1993 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the impulse of -2.5% in REER (dashed line), (iv) Predicted values after the impulse of +2.5% in REER (dash-dot line).

Table 18
Impulse response on WPI (±2.5% REER shock given in April 1993)

Month/Year	Model WPI before impulse	Predicted WPI after -2.5% REER shock	% variation	Predicted WPI after +2.5% REER shock	%variation
May, 93	236.90	236.93	0.01	236.87	-0.01
June, 93	239.58	239.61	0.01	239.54	-0.02
July, 93	242.31	242.33	0.01	242.28	-0.01
August, 93	244.66	244.65	-0.00	244.66	0.00
September, 93	246.35	246.33	-0.01	246.36	0.00
October, 93	248.11	248.09	-0.01	248.11	0.00
November, 93	249.68	249.67	-0.00	249.67	-0.00
December, 93	251.39	251.42	0.01	251.32	-0.03
January, 94	253.21	253.29	0.03	253.09	-0.05
February, 94	255.28	255.41	0.05	255.11	-0.07
March, 94	257.16	257.35	0.07	256.93	-0.09
April, 94	259.45	259.67	0.08	259.18	-0.10
May, 94	261.93	262.18	0.10	261.64	-0.11
June, 94	264.74	265.00	0.10	264.43	-0.12
July, 94	267.64	267.92	0.10	267.29	-0.13
August, 94	270.55	270.87	0.12	270.16	-0.14
September, 94	273.12	273.49	0.14	272.68	-0.16
October, 94	275.58	276.02	0.16	275.08	-0.18
November, 94	277.75	278.26	0.18	277.18	-0.21
December, 94	279.74	280.31	0.20	279.11	-0.23
January, 95	281.84	282.47	0.22	281.15	-0.24
February, 95	283.56	284.25	0.24	282.81	-0.26
March, 95	285.11	285.86	0.26	284.31	-0.28
April, 95	286.66	287.47	0.28	285.80	-0.30
May, 95	288.45	289.31	0.30	287.52	-0.32
June, 95	290.32	291.24	0.32	289.33	-0.34
July, 95	292.21	293.19	0.34	291.16	-0.36
August, 95	294.11	295.15	0.35	293.00	-0.38
September, 95	295.96	297.06	0.37	294.80	-0.39
October, 95	297.78	298.93	0.39	296.57	-0.41
November, 95	299.44	300.64	0.40	298.17	-0.42
December, 95	300.89	302.14	0.42	299.58	-0.44
January, 96	302.15	303.44	0.43	300.79	-0.45
February, 96	303.25	304.58	0.44	301.86	-0.46
March, 96	304.31	305.67	0.45	302.89	-0.47

The price behaviour in response to the shock given to the exchange rate in April 1994 depicts, more or less, a similar pattern (Figure 19, Table 19). The impact, in this case, seems to have become more pronounced relatively more quickly than in the case of the previous shock.

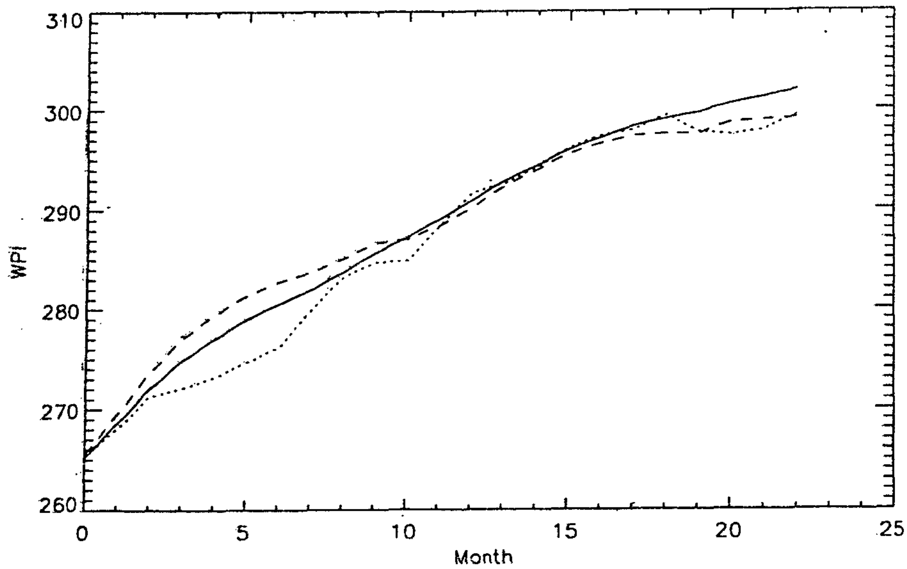


Figure 19 : The monthly values of variable WPI before and after the impulse is given in April 1994. The values range from May 1994 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the impulse of -2.5% in REER (dashed line), (iv) Predicted values after the impulse of +2.5% in REER (dash-dot line).

Table 19
Impulse response on WPI (±2.5% REER shock given in April 1994)

Month/Year	Model WPI before impulse	Predicted WPI after -2.5% REER shock	% variation	Predicted WPI after +2.5% REER shock	%variation
May, 94	265.21	265.25	0.02	265.18	-0.01
June, 94	268.66	268.74	0.03	268.58	-0.03
July, 94	272.06	272.22	0.06	271.90	-0.06
August, 94	274.90	275.13	0.08	274.66	-0.09
September, 94	277.06	277.38	0.12	276.72	-0.12
October, 94	279.01	279.42	0.15	278.58	-0.15
November, 94	280.54	281.05	0.18	280.02	-0.19
December, 94	281.97	282.56	0.21	281.37	-0.21
January, 95	283.76	284.41	0.23	283.09	-0.24
February, 95	285.67	286.38	0.25	284.94	-0.26
March, 95	287.27	288.05	0.27	286.48	-0.27
April, 95	289.07	289.92	0.29	288.22	-0.29
May, 95	290.95	291.85	0.31	290.03	-0.32
June, 95	292.87	293.84	0.33	291.89	-0.33
July, 95	294.37	295.41	0.35	293.32	-0.36
August, 95	295.96	297.06	0.37	294.84	-0.38
September, 95	297.17	298.35	0.40	295.98	-0.40
October, 95	298.31	299.54	0.41	297.06	-0.42
November, 95	299.04	300.33	0.43	297.73	-0.44
December, 95	299.64	300.97	0.44	298.30	-0.45
January, 96	300.55	301.94	0.46	299.15	-0.47
February, 96	301.21	302.65	0.48	299.74	-0.49
March, 96	301.93	303.45	0.50	300.40	-0.51

As observed in the previous paragraphs, an increase in money supply tends to be inflationary while an appreciation of REER would be deflationary. A policy shock, which combines a 5 per cent increase in money supply and a 2.5 per cent appreciation of REER should, going by the above logic, have counteracting effects on prices. In other words, a rise in M_3 would increase the price level while an REER appreciation would have a sobering effect on it. Both the opposing forces may thus be embedded in the price level. The outcomes of such a shock given in April 1993 have been plotted in Figure 20. It is evident from the graph that the combined shock has been found to be inflationary up to the 22nd month. If we compare, Table 20 with Table 17, we find that the impact of the combined shock on prices is somewhat lower than that of a single shock of +5% given to M_3. The lower impact on prices under the combined shock is attributed to the impact of the appreciation of REER, as expected. Further, as mentioned earlier, the impact of monetary shocks tapers off whereas that of REER accelerates. Therefore, it is possible for the combined effect to turn negative, once the monetary impact gets weakened. In fact, the combined effect was generally negative from the 23rd month.

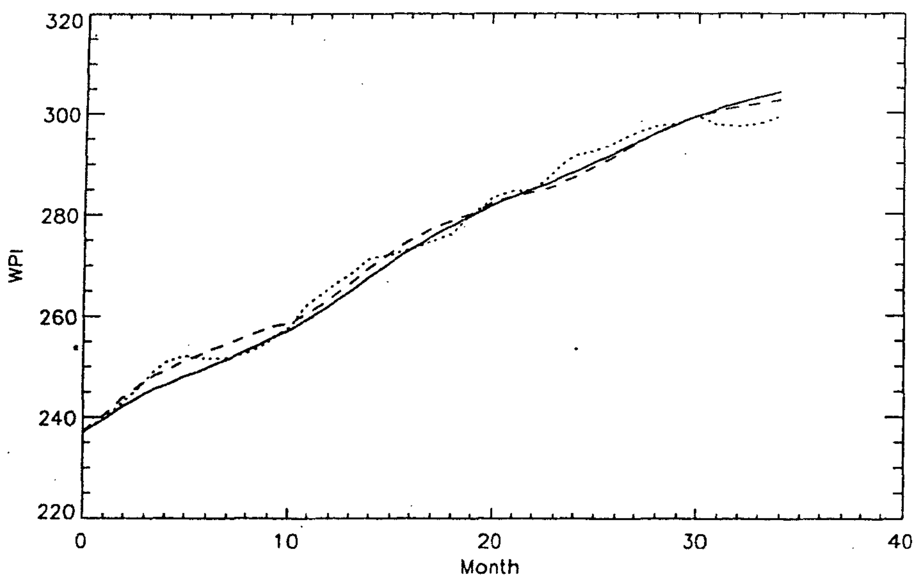


Figure 20 : The monthly values of variable WPI before and after the impulse is given in April 1993. The values range from May 1993 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the combined impulse of +5% in M_3 and +2.5% in REER (dashed line).

Table 20
Impulse response on WPI (combined shock of + 5% on M_3 and +2.5% on REER given in April 1993)

Month/Year	Model WPI before impulse	Predicted Values after impulse	% variation
May, 93	236.90	237.15	0.11
June, 93	239.58	240.45	0.36
July, 93	242.31	243.96	0.68
August, 93	244.66	247.09	0.99
September, 93	246.35	249.22	1.17
October, 93	248.11	251.23	1.26
November, 93	249.68	252.96	1.31
December, 93	251.39	254.46	1.22
January, 94	253.21	255.99	1.10
February, 94	255.28	257.76	0.97
March, 94	257.16	258.60	0.56
April, 94	259.45	260.58	0.44
May, 94	261.93	263.16	0.47
June, 94	264.74	266.31	0.59
July, 94	267.64	269.46	0.68
August, 94	270.55	272.48	0.71
September, 94	273.12	275.01	0.69
October, 94	275.58	277.10	0.55
November, 94	277.75	278.75	0.36
December, 94	279.74	280.16	0.15
January, 95	281.84	282.40	0.20
February, 95	283.56	283.58	0.01
March, 95	285.11	284.52	-0.21
April, 95	286.66	285.61	-0.37
May, 95	288.45	287.40	-0.36
June, 95	290.32	289.48	-0.29
July, 95	292.21	291.58	-0.22
August, 95	294.11	293.92	-0.06
September, 95	295.96	296.02	0.02
October, 95	297.78	297.96	0.06
November, 95	299.44	299.39	-0.02
December, 95	300.89	300.49	-0.13
January, 96	302.15	301.33	-0.27
February, 96	303.25	301.98	-0.42
March, 96	304.31	302.69	-0.53

The impact on prices was, by and large, similar when an identical combined shock was given in April 1994. For about first 10 months, the net effect was inflationary (Figure 21, Table 21). Thereafter, the simulated price level was below the model implying the weakening of the monetary impact on prices under the weight of the growing impact of exchange rate appreciation.

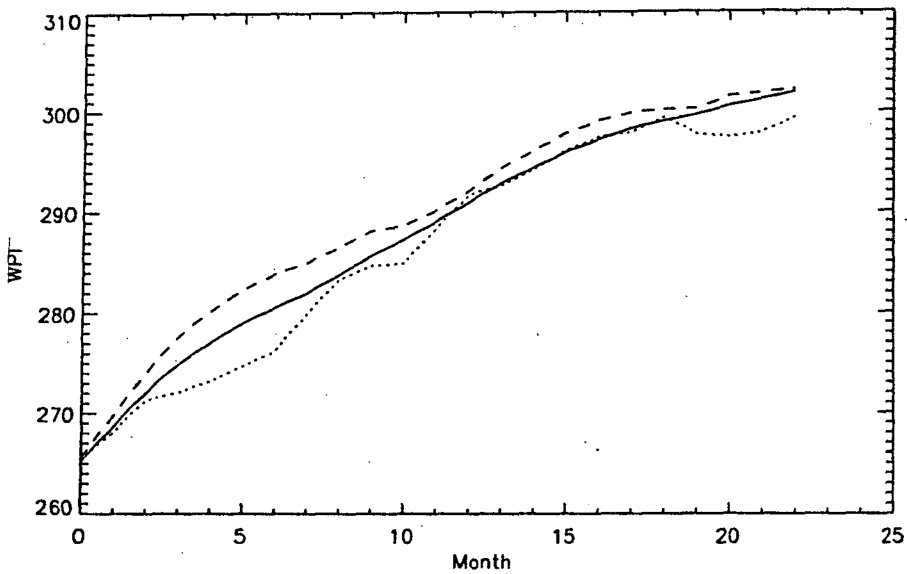


Figure 21 : The monthly values of variable WPI before and after the impulse is given in April 1994. The values range from May 1994 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the combined impulse of + 5% in M_3 and +2.5% in REER (dashed line).

Table 21
Impulse response on WPI (combined shock of +5% on M_3 and +2.5% on REER given in April 1994)

Month/Year	Model WPI before impulse	Predicted Values after impulse	% variation
May, 94	265.21	265.51	0.11
June, 94	268.66	269.57	0.34
July, 94	272.06	273.68	0.60
August, 94	274.90	277.12	0.81
September, 94	277.06	279.49	0.88
October, 94	279.01	281.39	0.85
November, 94	280.54	282.78	0.80
December, 94	281.97	283.75	0.63
January, 95	283.76	285.11	0.48
February, 95	285.67	286.66	0.35
March, 95	287.27	287.06	-0.07
April, 95	289.07	288.42	-0.22
May, 95	290.95	290.24	-0.24
June, 95	292.87	292.34	-0.18
July, 95	294.37	293.95	-0.14
August, 95	295.96	295.53	-0.15
September, 95	297.17	296.60	-0.19
October, 95	298.31	297.30	-0.34
November, 95	299.04	297.51	-0.51
December, 95	299.64	297.52	-0.71
January, 96	300.55	298.67	-0.63
February, 96	301.21	298.90	-0.77
March, 96	301.93	299.12	-0.93

Finally, a policy shock which combines an increase in money supply and fall in REER could be expected to be highly inflationary. This has been shown in Figures 22 and 23 and Tables 22 and 23. The predicted values after the shock remained above the model throughout the sample period both when the shock was administered in April 1993 and in April 1994.

Table 22
Impulse response on WPI (combined shock of + 5% on M_3 and -2.5% on REER given in April 1993)

Month/Year	Model WPI before impulse	Predicted Values after impulse	% variation
May, 93	236.90	237.22	0.14
June, 93	239.58	240.51	0.39
July, 93	242.31	244.00	0.70
August, 93	244.66	247.08	0.99
September, 93	246.35	249.20	1.16
October, 93	248.11	251.25	1.27
November, 93	249.68	253.06	1.35
December, 93	251.39	254.68	1.31
January, 94	253.21	256.35	1.24
February, 94	255.28	258.25	1.16
March, 94	257.16	259.23	0.80
April, 94	259.45	261.30	0.71
May, 94	261.93	263.96	0.78
June, 94	264.74	267.16	0.91
July, 94	267.64	270.38	1.02
August, 94	270.55	273.49	1.09
September, 94	273.12	276.14	1.11
October, 94	275.58	278.35	1.01
November, 94	277.75	280.12	0.85
December, 94	279.74	281.63	0.68
January, 95	281.84	283.98	0.76
February, 95	283.56	285.28	0.61
March, 95	285.11	286.32	0.42
April, 95	286.66	287.52	0.30
May, 95	288.45	289.43	0.34
June, 95	290.32	291.63	0.45
July, 95	292.21	293.86	0.56
August, 95	294.11	296.33	0.75
September, 95	295.96	298.55	0.88
October, 95	297.78	300.60	0.95
November, 95	299.44	302.14	0.90
December, 95	300.89	303.31	0.80
January, 96	302.15	304.21	0.68
February, 96	303.25	304.92	0.55
March, 96	304.31	305.68	0.45

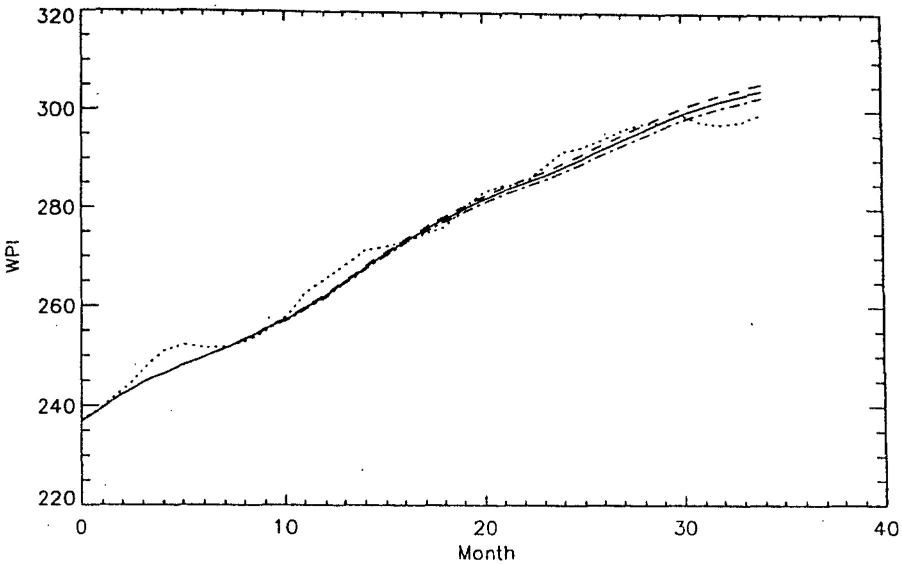


Figure 22 : The monthly values of variable WPI before and after the impulse is given in April 1993. The values range from May 1993 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the combined impulse of +5% in M_3 and -2.5% in REER (dashed line).

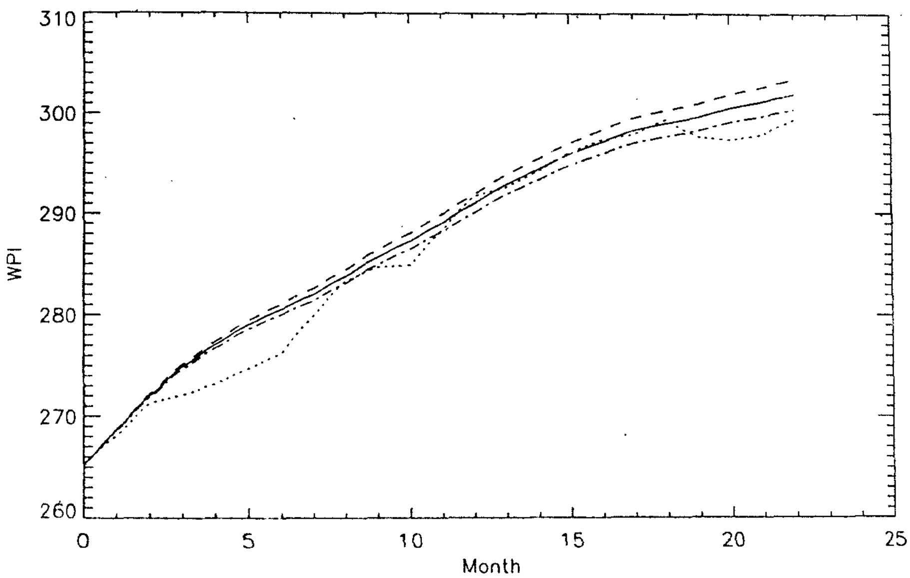


Figure 23 : The monthly values of variable WPI before and after the impulse is given in April 1994. The values range from May 1994 to March 1996. (i) Observed values before the impulse (dotted line), (ii) Model values before the impulse (solid line), (iii) Predicted values after the combined impulse of +5% in M_3 and -2.5% in REER (dashed line).

Table 23
Impulse response on WPI (combined shock of +5% on M_3 and -2.5% on REER given in April 1994)

Month/Year	Model WPI before impulse	Predicted Values after impulse	% variation
May, 94	265.21	265.58	0.14
June, 94	268.66	269.74	0.40
July, 94	272.06	274.00	0.71
August, 94	274.90	277.62	0.99
September, 94	277.06	280.19	1.13
October, 94	279.01	282.29	1.18
November, 94	280.54	283.88	1.19
December, 94	281.97	285.01	1.08
January, 95	283.76	286.50	0.97
February, 95	285.67	288.15	0.87
March, 95	287.27	288.67	0.49
April, 95	289.07	290.16	0.38
May, 95	290.95	292.11	0.40
June, 95	292.87	294.35	0.51
July, 95	294.37	296.11	0.59
August, 95	295.96	297.83	0.63
September, 95	297.17	299.04	0.63
October, 95	298.31	299.86	0.52
November, 95	299.04	300.18	0.38
December, 95	299.64	300.27	0.21
January, 96	300.55	301.52	0.32
February, 96	301.21	301.86	0.22
March, 96	301.93	302.20	0.09

The foregoing exercises give rise to a number of questions: Whether independent shocks alone can explain inflation dynamics in India? Whether a rise in REER can bring about a fall in prices and *vice versa?* What kind of combined policy shock the policy makers would have to avoid?

Increase in money supply would have no doubt raised the price level as evident from the money supply shock discussed earlier. This warrants a matching fall in nominal effective exchange rate so as to keep the real effective exchange rate stable. The factors preventing a downward adjustment of the nominal effective exchange rate might have resulted in an appreciation of the real effective exchange rate. However, the correction of real effective exchange rate beyond this point could be costly for the economy in two ways. First of all, increase in money supply may raise the general price level, thereby eroding export competitiveness and secondly, imports may be highly costly adding to the price increase.

Shock given to REER established the fact that depreciation of REER would be inflationary while its appreciation would be deflationary although REER changes affect prices after a time lag. In view of this, a stable REER would be consistent with the objective of price stability.

Since realised inflation is a product of several factors, individual shocks may not adequately explain the price behaviour in the economy. In case of combined shocks, prices were found to be adjusting almost concurrently, irrespective of the developments in the real sector. The combined shock essentially represents an area of interest for monetary authorities. Purchase of foreign currency from the market by the central bank leads to increase in reserve money which may have adverse repercussions on domestic prices, which in turn may require adjustment of exchange rate. Inflow of capital, *inter alia,* may prevent the depreciation of NEER for a while despite inflation differential between the domestic economy and the major trading partners. However, in the long run, the purchasing power parity would hold good. Therefore, it is not prudent to expand M_3 under the notion that appreciation of REER may have sobering effect on the prices.

The authorities need to be particularly vigilant to avoid a situation of expansionary M_3 combined with depreciation of REER. Such a combination is highly inflationary. However, within the objective of price stability, there exists a case for intervention up to a limit set by equilibrium rate of real exchange rate. The exchange rate adjustment beyond the said limit would tend to be inflationary as evidenced by the simulation results. It is, however, better to keep money supply under check so as to control price increases. Nevertheless, the domestic currency may still appreciate on account of large inflows of capital. In view of this, there is a need for greater coordination of monetary policy and exchange rate policy. If, in addition, fiscal restraint is rendered effective, the monetary authority would be in a better position to stabilise the exchange rate through intervention without facing severe policy conflict between the exchange rate stability and the price stability. Ultimately, it is the price stability that imparts stability to the exchange rate.

7. Summary and Conclusions

The basic objective of this study was to apply ANN methodology to construct dynamic models for macroeconomic variables and compare the results with the conventional approaches. The essential feature of ANN technique is that the model is generated from the data and requires weak assumptions. Further, the method is not restricted to modelling linear systems alone. Actually, even chaotic deterministic dynamics have been successfully modelled using the ANN approach. In this work, we have used monthly data of five variables - M_3, WPI, REER, IIP and FOODS for the period April 1975 to March 1996.

The time series data was pre-analysed in order to characterize the dynamics which generated the series. This analysis provided us with the values of time lag τ and the embedding dimension d. These parameters also enabled us to partially decide on the network architecture. Following this analysis, different studies were carried out. We first discuss univariate models. In this case we adopted a two step approach. In the first step, a model was constructed to describe the overall trend for each of the five economic time series. We found that the overall trends are well captured by the models. The second step models the residuals and we use the error corrected ANN models to make out of sample predictions. These predictions are compared with the conventional Box-Jenkins (BJ) (ARMA) models. The predictions for M_3, WPI (CPI) and REER are comparable in the two methods. The BJ model is better for the FOODS data.

Thus, so far as short term forecasts are concerned the conventional univariate BJ approach appears to be very satisfactory. It should be stressed, however, that conventional (BJ) approach does not generate a model for the overall dynamics. It essentially bypasses this question by considering suitable differences in the data.

We also carried out a number of multivariate studies having different objectives. An obvious one is to find non-linear relations between the variables. We find two such relations which are nonlinear in nature.

A second set of calculations consisted of constructing multivariate dynamic models for the trends. These models also reproduce the overall dynamic features well. Further, we modelled the residuals by evaluating the cross correlations between the variables and including the significant ones in the ANN framework. Out of sample forecasts with error correction were made and compared with the conventional multivariate regression analysis including the cointegrating relation. With the ANN approach, the short term predictions are somewhat better or comparable to those with VECM for M_3, WPI and IIP. ANN gives better predictions for REER whereas for FOODS the conventional VECM approach gives better results.

A third set of studies in the multivariate ANN framework involved giving impulses to M_3 and REER and following their influence in time. These results are discussed at length in Section 6. Clearly, these have immense value in terms of policy implications. The dynamics of inflation in India is adequately explained in terms of monetary factor. It is further borne out from the simulation exercise that the tempo of price changes dampens over time with a single dose of money supply shock. In other words, inflationary pressure cannot continue in the system indefinitely unless money supply is sustained at a higher level. Moreover, there is a need for correction of nominal exchange rate so as to keep the real effective exchange rate stable depending on inflation differential between the home country and the major trading partner. However, overcorrection was found to be harmful for the economy. Hence, coordination of monetary policy with exchange rate policies is necessary in the liberalised environment. It should be pointed out that only a fully dynamic model can carry out such studies. The traditional time series methods (VECM) are not useful for this purpose.

From our studies we have learnt that if the time series data is essentially linear then for the purpose of short term forecasts the usual BJ approach is very good and hence adequate. If the series has dominant nonlinear character then the BJ or VECM approach will not yield good results whereas the ANN would. However, for studying dynamics of the economic system, the ANN approach is the best data based methodology available and needs to be used along with appropriate economic theory.

Appendix I

Measures of Inflation in India

Inflation is one of the crucial macroeconomic indicators and an appropriate measurement of it is vital. In India, variations in the average price level are measured by three sets of indices, namely, (a) Wholesale Prince Index (WPI), (b) Implicit National Income Deflator, and (c) Consumer Price Indices (CPIs). There are differences in the compilation procedure, commodity coverage, weighting pattern, interval of release of data and geographical representation for each series. The use of each index number is often purpose-specific.

(a) The WPI is most frequently used to measure the overall inflation rate for all commodities and also for major groups, sub-groups and individual commodities. The WPI series is regularly compiled and published by the Office of the Economic Adviser, Ministry of Industry, on a weekly basis since 1942. The base year of WPI is revised generally at an interval of a decade after due consideration of all aspects by Technical Committees/Working Groups. The base year for WPI has been revised on four occasions in 1952-53, 1961-62, 1970-71 and in 1981-82. Weights are assigned to the commodities/sub-groups/major groups on the basis of the values of wholesale transactions at the time of changing the base year which remains valid till the next revision of the base year is made. The scope, coverage and weighting pattern, etc., of the current series (Base: 1981-82=100) are based on the recommendations of the Working Group headed by Dr. C. Rangarajan. The new series has significantly enlarged its coverage by including 447 distinct items as against 360 items in the 1970-71 series. The number of price quotations has also been increased to 2,371 in the new series. Compilation is continued to be done by using Laspeyre's Index, i.e., weighted average of price relatives (current price divided by base year's price) with base year's quantity as fixed weight. As the base year 1981-82 of the current series of WPI has become considerably old, the Government has appointed a Committee to initiate detailed exercise for the change of the base year.

(b) Unlike WPI, which captures the weekly rhythm of price changes, the implicit national income deflator is an annual series compiled by the Central Statistical Organisation (CSO) which is a ratio of GDP at current prices to GDP at constant prices. As it embraces all economic activities, the scope and coverage of the WPI cannot match with the national income deflator. In case of national income deflator, the relative shares (value added) of each sector (including services) of the economy in the GDP are implied weights which are significantly different from those assigned to each group in WPI. However, if the national income compilation is inadequate or weak, the GDP deflator would not be a suitable measure.

(c) The basic purpose of a consumer price index is to measure the changes in the level of retail prices of selected goods and services on which generally a homogeneous group of consumers spend major part of their income. The economic and social structure in India is so vast and differentiated that no single measure of consumer price index can even remotely depict the cost of living index of all classes. In India, therefore, three separate consumer price indices are computed for (i) industrial workers, (ii) urban non-manual employees, and (iii) agricultural labourers. The weighting patterns for all the series are broadly based on the results of the respective family budget surveys conducted from time to time. Unlike WPI, the CPI for Industrial Workers and for Urban Non-Manual Employees are compiled initially for selected industrial or urban centres and for agricultural labourers at the state level. They are aggregated to the all-India index as a weighted average of respective centre-wise or state-wise indices. Laspeyre's formula is used for the compilation of each index.

The CPI for Industrial Workers (CPI-IW) is compiled and published in every month by the Labour Bureau, Ministry of Labour, since 1950-51. The base year for CPI-IW has been revised on two occasions in 1960 and in 1982. The current series (Base: 1982 = 100) includes 260 items covering 70 selected industrial centres. The CPI for the Urban Non-Manual Employees (CPI-UNME) which depicts the changes of average retail prices of goods and services consumed by families having non-manual occupations in the non-agricultural sector, has been compiled by CSO since 1960. The base year is revised only once in 1984-85 on the basis of family living survey conducted during 1982-83 covering 59 selected centres.

The CPI for agricultural labourers (CPI-AL), which depicts the cost of living index of agricultural labourers at the state level has been compiled by the Labour Bureau, Ministry of Labour on a monthly basis since 1960-61. The base year has been changed only once from 1960-61 = 100 to 1986-87 = 100. One of the important uses of CPI-AL is the fixation as well as revision of the minimum wages for agricultural labour.

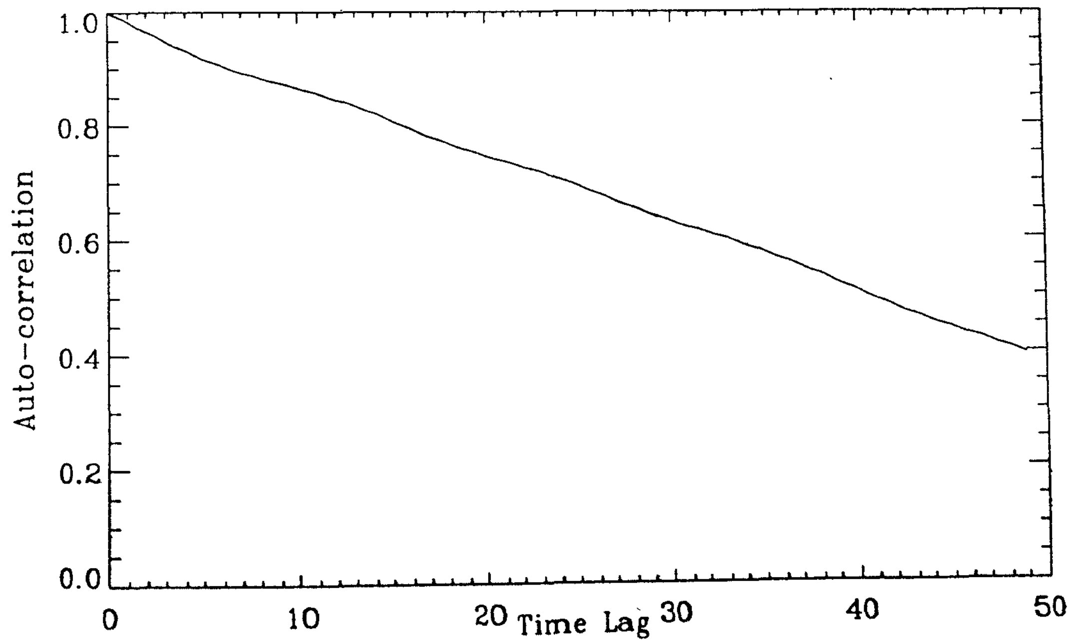
Appendix II

Characterization of Time Series Data

As emphasized earlier, before modelling the time series, it is necesary to ascertain the nature of the series through preliminary analysis. It also helps to partially decide the architecture of the network, if one is using artificial neural network framework for modelling. In general, it is worth examining the four plots viz., (i) Auto-correlation function, (ii) Average mutual information viz., time lag, (iii) Number of nearest false neighbours vs. dimension, and (iv) DVS plot. In combination, they give fairly good idea about the qualitative nature of the series. We illustrate their usage by applying them to monthly time series of REER.

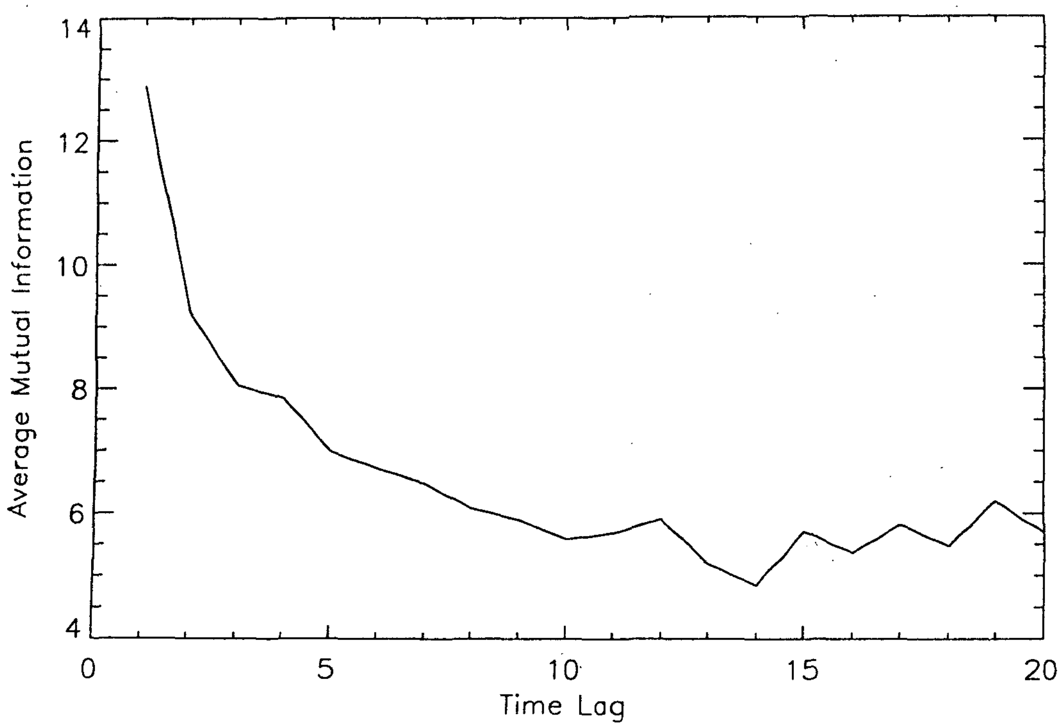
Figure 1 shows the auto-correlation function of the REER series. The fact that the auto-correlation function falls very slowly indicates that the coherence time is long and the generating process underlying the series is most likely deterministic.

Figure 1
Auto-correlation Function for REER



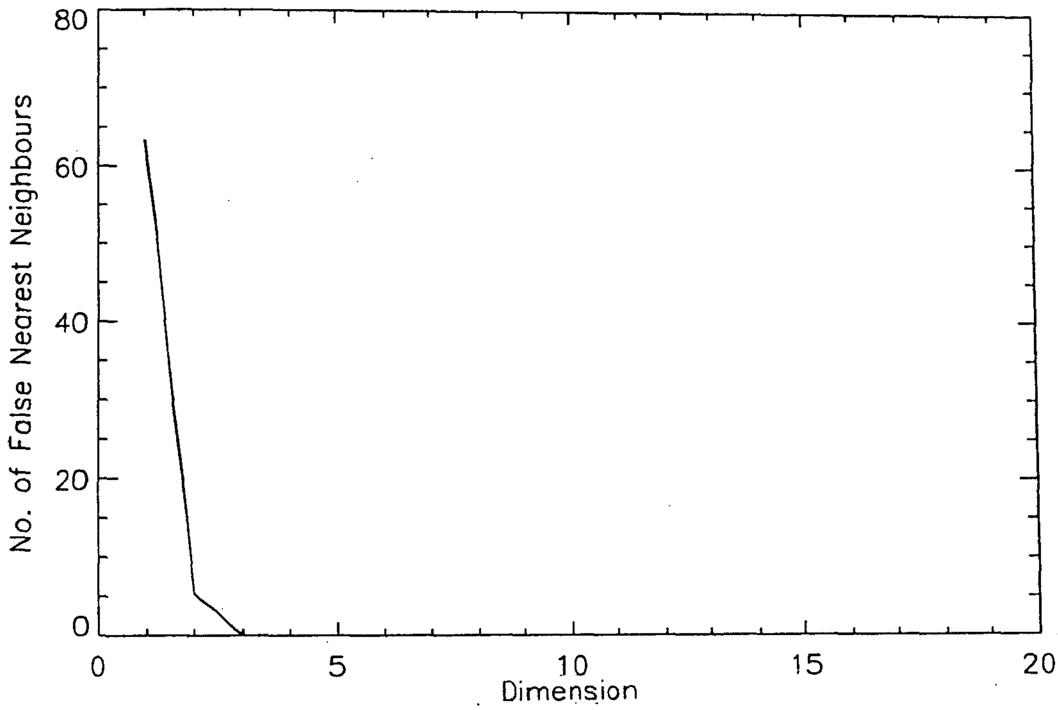
We next study in Figure 2 the average mutual information as a function of time lag. The quantity of average mutual information, as stated in the text, is a nonlinear version of auto-correlation and hence does take into account nonlinear correlations. The plot again shows slow fall. As per our criterion the time lag τ corresponds to first minimum in the plot. It may be noted here that the minimum is quite shallow. However, this situation is typical of all time series which have long coherence time.

Figure 2
Average Mutual Information Plot for REER



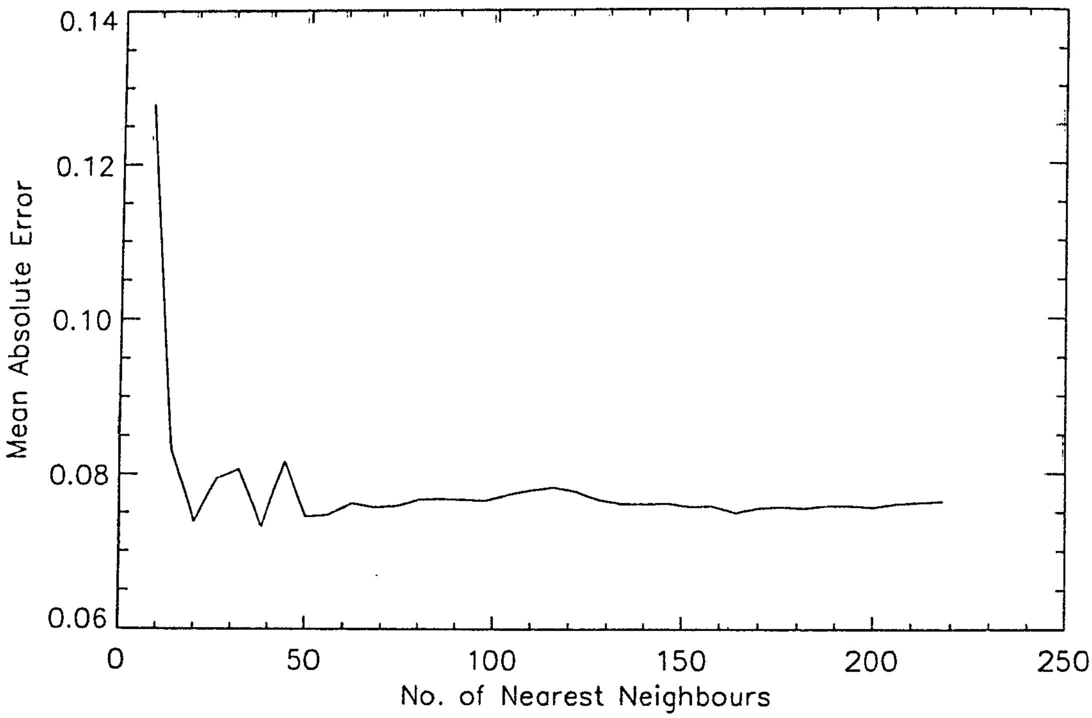
Using the time delay τ, our next objective is to determine the dimension d of the embedding space in which the original dynamics in phase space is reconstructed. Using the method of delay we first obtain the embedding space of different dimension and then evaluate the number of false nearest neighbours as we go from one dimension to the next. This is shown in Figure 3. The value of dimension at which the number of false nearest neighbours is zero or minimum is the appropriate value of embedding dimension d.

Figure 3
Plot of False Nearest Neighbours for REER



In Figure 4 we show the DVS plot in which the mean absolute error in the out-of-sample set is given as a function of number of nearest neighbours used for fitting the training set with linear predictor model. In DVS plot, the region of small neighbourhood is considered as deterministic and that of large neighbourhood is considered as stochastic. If the error increases as number of neighbours increases, the generating process is considered as nonlinear. If it decreases, then it is linear. The interpretation depends both on the nature of plot as well as on where the point of smallest error lies. The plot in Figure 4 clearly indicates that the generating process is low-dimensional deterministic with weak nonlinearity.

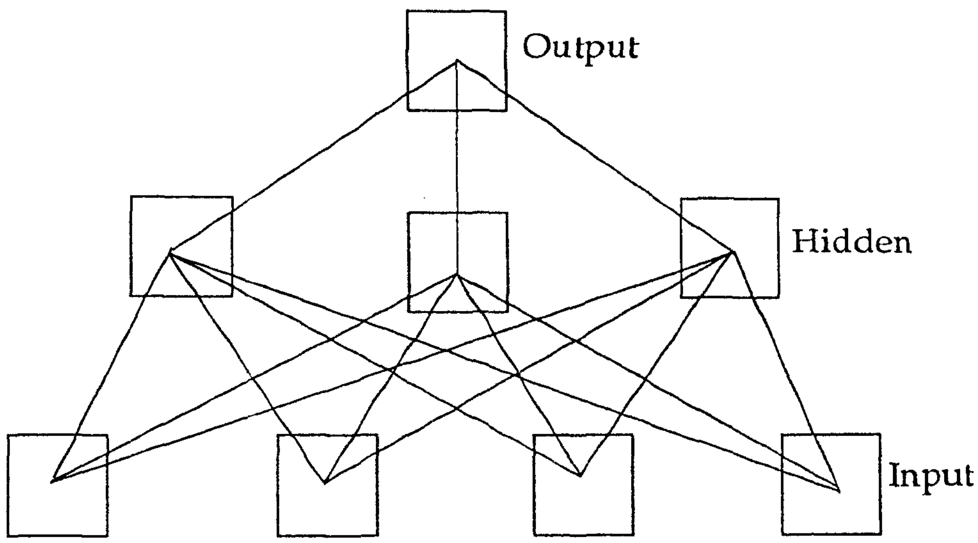
Figure 4
DVS Plot for REER (d=3, T=1, tau=10)



Appendix III

Artificial Neural Network (ANN) - Back Propagation

Consider a network with $n1$, $n2$ and $n3$ neurons in the input, hidden and output layers respectively.



Let,

Y1(i)	:	i=1,...n1 input data
O(k)	:	k=1,...n3 output data
W2(j,i)	:	Weight betweeen input layer neuron i and hidden layer neuron j
$W3(k,j)$	:	Weight between hidden layer neuron j and output layer neuron k

The input X2(j) to the neuron j in the hidden layer is taken to be the weighted sum of all the inputs. Thus, when there is full connectivity,

$$X2(j) = \sum_{i} W2(j,i)\ Y1(i) \qquad \begin{array}{l} i=1,\dots n1 \\ j=1,\dots n2 \end{array} \tag{A1}$$

The output of neuron j of the hidden layer is given by

$$Y2(j) = f(X2(j)) \tag{A2}$$

where f is the transfer function. For a continuous variable it is taken to be either a sigmoid $f(x) = \dfrac{1}{1+e^{-x}}$ or a hyperbolic tangent function $f(x) = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$.

Next one determines the input to the neuron k of the output layer from the neurons in the hidden layer.

$$X3(k) = \sum_{j} W3(k,j)\ Y2(j) \qquad \begin{array}{l} j=1,\dots n2 \\ k=1,\dots n3 \end{array} \tag{A3}$$

The output of neuron k of the output layer is

$$Y3(k) = f(X3(k)) \tag{A4}$$

The weights $W2(j,i)$ and $W3(k,j)$ are unknown and they form the complete set $(n_1 * n_2 + n_2 * n_3)$ of parameters of the model. These parameters are obtained by minimizing the cost function with respect to $W2$ and $W3$.

$$C = \frac{1}{2} \sum_{k=1}^{n3} [O(k) - Y3(k)]^2 \tag{A5}$$

In practice, one starts from random weights and evaluates the function C. In the next iteration the weights are corrected by amounts shown in Eqs. (A6)

$$\delta W3(k,j) = e2 \frac{\partial C}{\partial W3(k,j)}$$

$$\delta W3(j,i) = e1 \frac{\partial C}{\partial W2(j,i)} \tag{A6}$$

where e1 and e2 are parameters called learning rates. This iterative procedure is repeated until one has convergence. Essentially, it determines the weights which minimize the cost function from the data.

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