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Forecasting Inflation and IIP Growth: Bayesian Vector Autoregressive Model

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Maintaining low and stable inflation with sustainable growth is the prime objective of any monetary authority. To achieve the prime goal, reliable forecast of macroeconomic variables play an important role. In this paper, the authors tried to develop a forecasting model for inflation as well as IIP growth in a multivariate time series Bayesian framework, known as Bayesian Vector Autoregressive (BVAR) Model. The main advantage of using this model is the incorporation of prior information which may boost the forecasting performance of the model. Using the quarterly data on WPI, M1 and IIP during the period of first quarter of 1994-95 (Q1: 1994-95) to last quarter of 2007-08 (Q4: 2007-08), a VAR was developed and subsequently using Minnesota prior or Litterman's prior proposed by Litterman in 1980, a BVAR model was developed. Based on the comparison of forecasting performance of VAR and BVAR model, measured in terms of out-of-sample percentage root mean square error, it is found that BVAR model performed better than VAR model in case of inflation as well as IIP growth forecast.

JEL Classification : C1, C3, E2, E3.
Keywords : Inflation, Output, VAR, Bayesian VAR, Minnesota prior.

Introduction

Undoubtedly, maintaining inflation at low and stable rate which bust production environment without hearting common people is primary goal of any monetary authority at the globe. In the process of achieving this prime goal, while making monetary policy, a lot of information on monetary and fiscal variables are required. Along

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with these inputs, the reliable forecasts of macro-economic variables undoubtedly have significant policy implications. It is therefore, the search for better forecasting techniques to get reliable forecast is always vital. To access the general price situation, which is likely to be appeared in the next coming few months, the frequently used forecasting models are univariate time series models like autoregressive model, moving average model, autoregressive moving average model, etc. or multivariate times series model. The merit of using multivariate time series model is, along with incorporating past information of the target variable, it allows to incorporate inter-temporal interdependence of other variables for improving the forecasting performance. The commonly used multivariate time series model is vector autoregressive (VAR) model. But, the major setback of this model is the problem of overparameterisation. By the nature of the model, it requires to estimate large number of parameters which leads to large standard error. So, if some restriction can be imposed on the parameters then the performance of the model should be improved.

The facility of imposing restrictions is available in Bayesian Statistics by the way of prior information on parameters/coefficients. As, the name itself says about prior information, it is the information about the parameters which come before the experiment, by the way of other experiments, personal belief of the forecaster, etc., and then assigning probability distribution to each coefficients of the model. This Bayesian VAR (BVAR) approach provides more accurate forecasts (Litterman (1980), Kinal and Ratner (1986)). BVAR is also superior to VAR since it is robust to the choice of national variables, even when misspecified national variables are included (Shoesmith, 1990). Hence, a modified VAR restricting certain parameters is sometimes preferred. In general, the prior being used for BVAR is Minnesota prior or Litterman's prior proposed by Litterman in 1980. Some important studies being done using Bayesian VAR are for Minnesota (Litterman, 1980), New York state (Kinal and Ratner 1986), Texas (Gruben and Long 1988), Louisiana (Gruben and Hayes 1991), Iowa (Otrok and Whiteman 1998) and Philadelphia Metropolitan Area (Crone and McLaughlin 1999).

In this project, the idea is to develop a Bayesian Vector Autoregressive (BVAR) model for Indian Economy by allowing possibility of interactions between the important macroeconomic variables. Here, at the first stage, we have developed a VAR model for the two most important macroeconomic variables *viz.* industrial output growth and inflation of the economy. Next, the VAR model is modified to make a BVAR model. Lastly, a comparative study between the VAR and BVAR models is done based on the out-of-sample forecasting performance.

The rest of the paper is organized as follows. Section I gives the literature review. A short description on VAR process and BVAR process and on Litterman's prior is given in Section II. Section III presents the data, variables used and period of study. Section IV describes the empirical results. Finally, the concluding remarks are presented in Section V.

Section I
Literature Review

The comparative analysis of short-run forecasting methods used in this present work has been recurrent in the econometric literature. Three main trends were then distinguished. In the 1950s, the first forecasts were released to analyze business cycles and enlighten public decisions. As these were successfully used in the '60s and the '70s, forecasting developed later mainly through macroeconometric models, which were carried out by many economists (Mincer and Zarnowitz (1969), Makridakis & Hibon (1979), Fair (1979), Fonteneau (1982), Bodkin, Klein & Marwah (1990)). However, critics arose in the late '70s, (Lucas (1970), Kydland & Prescott (1977), Sims (1980)), saying that the forecasts were inaccurate and unable to anticipate the big crises of seventh and eighth decades of twentieth century. This period put an end to the golden age of forecasting based on econometric models and favoured the emergence of new methodological approaches.

Some works broke up with the traditional approach by offering diverse methods to study time series data (Kaman filter, Box-Jenkins methodology, the VAR modelling, state-space models). At the end of the '80s, empirical studies on these methods flourished, questioning their effectiveness and performances faced with the macroeconomic models (Kling and Blesser (1985), McNees (1986), Makridakis (1986), Wallis (1989), Aoki (1990). In short, this second trend showed that the methods based on time series data gave comparable or even superior results to the traditional macreconometric models.

The third trend corresponds to the present time. It started with questions about the non-stationarity of the series and their long-run evolution. The answers to these questions aroused a tremendous interest in econometric research. It consequently led to a large diversity of works on economic variables in 1990s. It is however too soon to measure the effectiveness and significance of these current methods, which remain to be improved.

The use of VAR models has been recommended by Sims (1980) as an efficient alternative to verify causal relationships in economic variables and to forecast their evolution. On the theoretical level, this approach has its foundation in the work of Wold (1938), Box and Jenkins (1980) and Tiao and Box (1981). Given the vector of variables, the classical VAR model explains each variable by its own past values and the past values of all other variables by a well-defined relation. For macroeconomic forecasting, VAR has become a standard tool. VARs produce dynamic forecasts that are consistent across equations and forecast horizons.

The issue which has entailed for a long time the controversy between the supporters and detractors of the Bayesian procedure is the estimation of the parameters of a model, either by using the statistical inference techniques or, on the contrary, by taking into account the previous knowledge of the economic system. The application of this procedure implies that an a priori probability has to be chosen and it can only be applied to models with a finite number of parameters. Yet, since most of the macroeconomic variables are from stochastic tendencies, the specification of their distribution turns out to be necessary. Usually, the hypothesis of normality for the coefficients is adopted since, in most cases, the underlying economic theory has little influence on the distribution of errors. In the field of multivariate modelling, Litterman suggested the use of the Bayesian procedure (1980) as an efficient way of avoiding some of the problems posed by Sims VAR models. The over-parametrisation is mainly the cause of these problems. Indeed, even if the reduced-size systems are involved, too many parameters have to be considered, which turns out to be non-significant after applying the hypothesis tests. Thus, it is necessary to put forward that the out-of-sample forecasts obtained by means of a standard VAR model depend a lot on the number of lags, even though the values observed and calculated are very close on the estimation period. In order to bypass these difficulties, Litterman (1980) introduces some a-priori knowledge in the formulation of his model by means of a distribution of probabilities.

The primary focus of monetary policy, both in India and elsewhere, has traditionally been the maintenance of a low and stable rate of aggregate price inflation along with sustainable economic growth. The underlying justification for this objective is the widespread consensus supported by numerous economic studies that inflation is costly insofar as it undermines real, wealth-enhancing economic activity. If anything, this consensus is probably stronger today than it ever has been in the past. Indeed, it could be argued that much of the improvement in Indian living standards which has taken place over the last two decades would not have been achieved without the establishment of a credible low inflation environment.

This paper focuses mainly on BVAR models. Over the past twenty years, the BVAR approach has gained widespread acceptance as a practical tool to provide reasonably accurate macroeconomic forecasts when compared to conventional macroeconomic models or alternative time series approaches.

Section II
Methodology: An Overview

Vector Autoregressive Model:

In notational form, mean-adjusted VAR(p) model (VAR model of order p) can be written as

$$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \text{ ; t=0,1,2,3,....}$$

Where, $y_t = (y_{1t}, ..., y_{Kt})$ is a (K×1) random vector, the A_i are fixed (K×K) coefficient matrices and $u_t = (u_{1t}, ..., u_{Kt})$ is a K-dimensional white noise or innovation process, i.e., E(u_t)=0, $E(u_t u_t^{'}) = \Sigma_u$ and $E(u_t u_s^{'}) = 0$ for s≠ t.

The same can be written for t=1,……,T, compactly as

$$Y = AX + U$$

or $y = (X^{'} \otimes I_K)\beta + u$

where,

$$Y = (y_1, ..., y_T)_{(K \times T)}$$

$$A = (A_1, ..., A_p)_{(K \times Kp)}$$

$$X = (Y_0, ..., Y_{T-1})_{(Kp \times T)}$$

$$Y_t = \begin{bmatrix} y_t \\ . \\ . \\ . \\ y_{t-p+1} \end{bmatrix}_{(Kp \times 1)}$$

$$y = vec(Y)_{(KT \times 1)}$$

$$\beta = vec(A)_{(K^2 p \times 1)}$$

$$U = (u_1, ..., u_T)_{(K \times T)}$$

$$u = vec(U)_{(KT \times 1)}$$

Under the standard VAR, coefficient vector β is unknown but fixed which has to be estimated.

Bayesian Vector Autoregressive(BVAR) Model:

On the other hand, in BVAR model the coefficients β's are considered as variables which some known distribution known as prior distribution. The parameter of prior distribution is known as hyperparameter. In this project, we have used Minnesota prior or Litterman's prior proposed by Litterman in 1980. Under this prior, parameter vector β has a prior multivariate normal distribution with known mean β* and covariance matrix V_β, hence the prior density is written as

$$f(\beta) = (\frac{1}{2\pi})^{K^2 p} \left|V_\beta\right|^{-1/2} \times \exp[-\frac{1}{2}(\beta - \beta^*)V_\beta^{-1}(\beta - \beta^*)^{'}]$$

Whereas, the likelihood function for the Gaussian process becomes

$$l(\beta| \, y) = (\frac{1}{2\pi})^{KT/2} \left|I_T \otimes \Sigma_u\right|^{-1/2} \times$$

$$\exp[-\frac{1}{2}(y-(X \otimes I_K)\beta)(I_T \otimes \Sigma_u)^{-1}(y-(X \otimes I_K)\beta)^{'}]$$

Therefore, using Bayes' theorem,

$$f(\beta / y) = \frac{l(\beta / y) f(\beta)}{\int l(\beta / y) f(\beta) d\beta}$$

the posterior density is written as

$$l(\beta| \, y) \propto \exp[-\frac{1}{2}(\beta - \overline{\beta})\overline{\Sigma_\beta}^{-1}(\beta - \overline{\beta})^{'}]$$

where the posterior mean is

$$\overline{\beta} = [V_\beta^{-1} + (X'X \otimes \Sigma_u^{-1})]^{-1}[V_\beta^{-1}\beta^* + (X' \otimes \Sigma_u^{-1})y]$$

and the posterior covariance matrix is

$$\overline{\Sigma_\beta} = [V_\beta^{-1} + (X'X \otimes \Sigma_u^{-1})]^{-1}$$

In practice, the prior mean β* and the prior variance V_β need to be specified. According to Litterman, the prior variance can be given by

$$\upsilon_{ij}(l) = \begin{cases} (\lambda / l)^2 & if \ \text{i=j} \\ (\lambda\theta\sigma_{ii} / l\sigma_{jj})^2 & if \ \text{i≠j} \end{cases}$$

where $\upsilon_{ij}(l)$ is the prior variance of the $(i,j)^{th}$ element of A_l, λ is the prior standard deviation of the diagonal elements of A_l, θ is a constant in the interval (0,1), and σ_{ii}^2 is the ith diagonal element of Σ_u .

Section III
Selected of Variables and Time Period of the Study

In India, the measurement of general price situation of the country as a whole is based on Wholesale price Index (WPI), we have used following variables in our study:

- Wholesale Price Index (WPI)
- Index of Industrial Production (IIP), and
- Narrow Money (M1).

We have used quarterly data during the period of first quarter of 1994-95 (Q1: 1994-95) to last quarter of 2007-08 (Q4: 2007-08). The model is fitted based on data till Q4: 2006-07, whereas, data for remaining period is being used for testing the model performance.

Before developing the model logarithmic transformation has been used. Whereas, to adjust with seasonality, seasonal dummies are used.

Section IV
Empirical Analysis

Stationary: The Augmented Dickey Fuller test is used for testing stationarity at the level and at first difference. Based on the results of the test statistics (Appendix 1), it can be observed that the variables are seems to be stationary at first difference.

Selection of Order of VAR: For the purpose of selecting order of VAR, the Minimum Information Criteria as well as Univariate Model White Noise Diagnostics are being used and based on these criteria, the order of VAR is found to be two.

The results of VAR(2) is given in the Appendix 2. Based on these results, it can be observed that the diagnostics results of this model are appears to be satisfactory. And out of sample percentage root mean

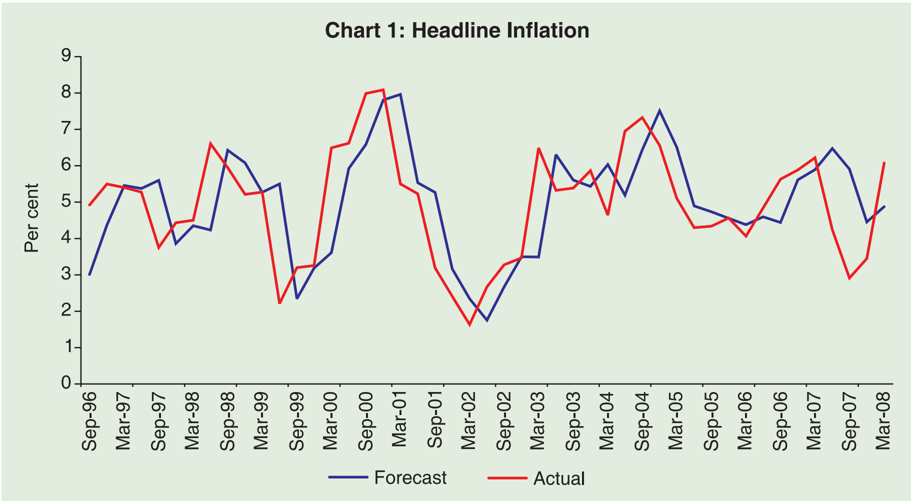
Table 1: Comparison of the Models

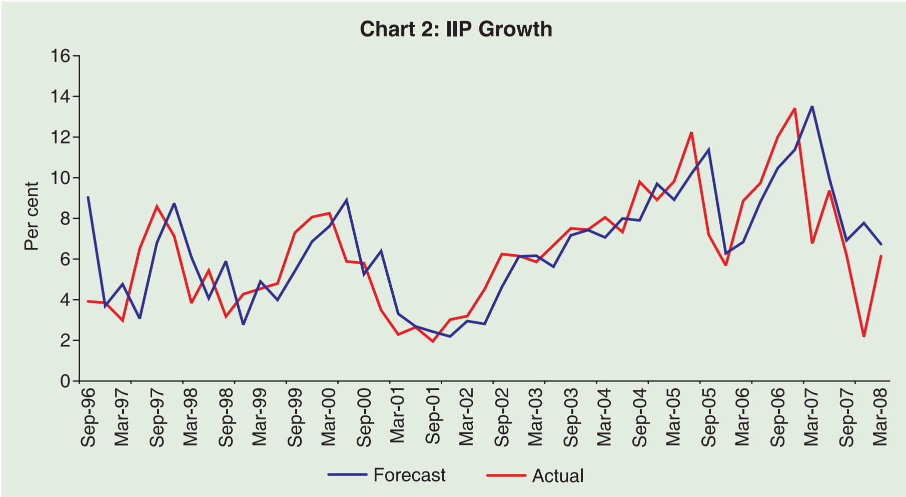
Model	Out of Sample PRMSE	
	WPI	IIP
VAR(2)	1.4932	4.2508
BVAR(2)	1.4400	3.6055

square error (PRMSE) for WPI for four quarters is 1.4932 per cent, whereas, for IIP, it is 4.2508 per cent.

Selection of values of lambda and theta in Litterman prior: Here, since the VAR model is developed at difference of the variables, therefore, the absolute value of the parameters would be less then one and hence mean of prior distribution is taken as zero, whereas, the degree of closeness of parameters to the prior mean can be controlled by suitable values of lambda and theta. Further, for selecting the a suitable values for lambda and theta, we have tried various combination for these parameters between 0 to 1 and based on PRMSE (Appendix 3), we found that lambda=0.3 and theta=0.9 are suitable values for BVAR(2). Therefore, BVAR(2) with lambda=0.3 and theta=0.9 was fitted and results of the model is given in Appendix 4.

From the results, given in the table 1, it can be observed that, out of sample PRMSE has been reduced while using BVAR in both the cases i.e. for WPI as well as IIP.





Section V
Concluding Remarks

In this project, with the objective of getting better forecast of inflation as well as IIP growth, quarterly data on WPI, IIP and M1 since first quarter of 1994-95 to fourth quarter of 2007-08 were used and we developed a VAR as well as Bayesian VAR (BVAR) model for forecasting the target variables. Further, based on the comparison of performance of these two models, it is found that the forecasting performance, measured in terms of out-of-sample percentage root mean square error of VAR model being used for forecasting inflation as well as IIP growth, has improved by applying Bayesian technique.

Appendix 1: Unit Root Test
Augmented Dickey-Fuller Unit Root Tests (Level)

Variable: lwpi

Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	1	0.1238	0.7069	5.23	0.9999
	2	0.1205	0.7060	5.55	0.9999
	3	0.1175	0.7052	5.63	0.9999
	4	0.1118	0.7038	4.08	0.9999
Single Mean	1	-0.3863	0.9315	-0.86	0.7923
	2	-0.2066	0.9428	-0.62	0.8555
	3	-0.1395	0.9466	-0.56	0.8704
	4	0.0838	0.9581	0.31	0.9765
Trend	1	-44.0181	<.0001	-4.98	0.0010
	2	-42.7256	<.0001	-4.01	0.0148
	3	-34.3860	0.0005	-3.27	0.0827
	4	-82.5674	<.0001	-3.24	0.0892
Variable: liip					
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	1	0.1238	0.7069	5.23	0.9999
	2	0.1205	0.7060	5.55	0.9999
	3	0.1175	0.7052	5.63	0.9999
	4	0.1118	0.7038	4.08	0.9999
Single Mean	1	-0.3863	0.9315	-0.86	0.7923
	2	-0.2066	0.9428	-0.62	0.8555
	3	-0.1395	0.9466	-0.56	0.8704
	4	0.0838	0.9581	0.31	0.9765
Trend	1	-44.0181	<.0001	-4.98	0.0010
	2	-42.7256	<.0001	-4.01	0.0148
	3	-34.3860	0.0005	-3.27	0.0827
	4	-82.5674	<.0001	-3.24	0.0892
Variable: lm1					
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	1	0.1374	0.7101	6.13	0.9999
	2	0.1319	0.7087	7.19	0.9999
	3	0.1262	0.7073	8.90	0.9999
	4	0.1222	0.7062	3.28	0.9996
Single Mean	1	0.3323	0.9688	0.73	0.9917
	2	0.4188	0.9719	1.47	0.9990
	3	0.4760	0.9737	3.33	0.9999
	4	0.5584	0.9762	2.55	0.9999
Trend	1	-9.3281	0.4442	-1.57	0.7891
	2	-0.5215	0.9915	-0.14	0.9927
	3	2.9137	0.9999	1.73	0.9999
	4	1.0159	0.9985	0.35	0.9984

Augmented Dickey-Fuller Unit Root Tests (Difference=1)

Variable: lwpi

Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	1	-16.6621	0.0028	-2.95	0.0040
	2	-9.9214	0.0245	-2.19	0.0285
	3	-3.4208	0.1989	-1.57	0.1090
	4	-2.2352	0.3006	-1.20	0.2062
Single Mean	1	-99.6940	0.0004	-6.79	0.0001
	2	-1124.78	0.0001	-6.42	0.0001
	3	-114.865	0.0001	-4.43	0.0008
	4	-316.520	0.0001	-3.93	0.0036
Trend	1	-100.378	0.0001	-6.75	<.0001
	2	-1321.11	0.0001	-6.37	<.0001
	3	-106.584	0.0001	-4.29	0.0072
	4	-241.817	0.0001	-3.84	0.0232
Variable: liip					
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	1	-244.186	0.0001	-11.26	<.0001
	2	623.2370	0.9999	-7.67	<.0001
	3	-2.7100	0.2542	-1.23	0.1977
	4	-3.4941	0.1940	-1.64	0.0946
Single Mean	1	-389.858	0.0001	-14.13	0.0001
	2	115.7601	0.9999	-18.48	0.0001
	3	-20.2941	0.0053	-2.61	0.0973
	4	-38.5153	0.0004	-3.23	0.0241
Trend	1	-388.594	0.0001	-13.97	<.0001
	2	116.0946	0.9999	-18.38	<.0001
	3	-21.3430	0.0283	-2.67	0.2548
	4	-39.1565	<.0001	-3.43	0.0602
Variable: lm1					
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau
Zero Mean	1	-21.4662	0.0005	-3.06	0.0029
	2	-10.7733	0.0187	-2.12	0.0336
	3	-1.0953	0.4525	-0.77	0.3787
	4	-0.4935	0.5672	-0.51	0.4905
Single Mean	1	-157.578	0.0001	-8.26	0.0001
	2	216.4850	0.9999	-8.91	0.0001
	3	-33.0310	0.0004	-3.03	0.0387
	4	-14.4477	0.0321	-2.12	0.2388
Trend	1	-166.277	0.0001	-8.48	<.0001
	2	181.8394	0.9999	-10.19	<.0001
	3	-77.2359	<.0001	-4.01	0.0148
	4	-41.7384	<.0001	-3.37	0.0680

Appendix 2: VAR (2)

Estimation Method: Least Squares Estimation

Seasonal Constant Estimates				
Variable	Constant	Season 1	Season 2	Season 3
dlwpi	0.01394	-0.01258	-0.01463	-0.00450
dliip	0.04654	-0.00519	0.00039	-0.13780
dlm1	0.01144	0.03351	0.05328	0.00301

Model Parameter Estimates

Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
dlwpi	CONST1	0.00158	0.00961	0.16	0.8698	1
	SD_1_1	0.00144	0.01299	0.11	0.9124	S_1t
	SD_1_2	-0.00197	0.01170	-0.17	0.8671	S_2t
	SD_1_3	0.00613	0.01194	0.51	0.6096	S_3t
	AR1_1_1	0.22639	0.10926	2.07	0.0426	dlwpi(t-1)
	AR1_1_2	0.05563	0.08592	0.65	0.5199	dliip(t-1)
	AR1_1_3	0.08854	0.06744	1.31	0.1943	dlm1(t-1)
	AR2_1_1	-0.16897	0.09405	-1.80	0.0775	dlwpi(t-2)
	AR2_1_2	0.05573	0.07045	0.79	0.4321	dliip(t-2)
	AR2_1_3	0.11449	0.05616	2.04	0.0460	dlm1(t-2)
dliip	CONST2	0.02409	0.01212	1.99	0.0515	1
	SD_2_1	0.00772	0.01653	0.47	0.6422	S_1t
	SD_2_2	0.03394	0.01469	2.31	0.0244	S_2t
	SD_2_3	-0.09845	0.01499	-6.57	0.0001	S_3t
	AR1_2_1	-0.41799	0.13353	-3.13	0.0027	dlwpi(t-1)
	AR1_2_2	0.20349	0.10809	1.88	0.0647	dliip(t-1)
	AR1_2_3	0.13000	0.08384	1.55	0.1264	dlm1(t-1)
	AR2_2_1	-0.00292	0.11195	-0.03	0.9793	dlwpi(t-2)
	AR2_2_2	-0.06808	0.09121	-0.75	0.4584	dliip(t-2)
	AR2_2_3	0.15537	0.06984	2.22	0.0299	dlm1(t-2)
dlm1	CONST3	0.00490	0.01588	0.31	0.7589	1
	SD_3_1	0.03024	0.02122	1.42	0.1595	S_1t
	SD_3_2	0.05074	0.01916	2.65	0.0104	S_2t
	SD_3_3	0.05399	0.01948	2.77	0.0075	S_3t
	AR1_3_1	-0.17080	0.17574	-0.97	0.3351	dlwpi(t-1)
	AR1_3_2	0.07945	0.14032	0.57	0.5734	dliip(t-1)
	AR1_3_3	-0.27610	0.11252	-2.45	0.0171	dlm1(t-1)
	AR2_3_1	-0.06520	0.14718	-0.44	0.6594	dlwpi(t-2)
	AR2_3_2	-0.14172	0.11526	1.23	0.2237	dliip(t-2)
	AR2_3_3	0.11877	0.09635	1.23	0.2226	dlm1(t-2)

Schematic Representation of Cross Correlations of Residuals

Variable/													
Lag	0	1	2	3	4	5	6	7	8	9	10	11	12
dlwpi	+..	...	...	...	...	...	...	...	...	...	...	...	...
dliip	.+.	...	...	...	...	...	...	...	...	...	...	...	...
dlm1	..+	...	...	...	...	...	...	...	...	...	...	...	...
	+ is > 2*std error, - is < -2*std error, . is between												

Portmanteau Test for Cross Correlations of Residuals

Up to Lag	DF	Chi-Square	Pr > ChiSq
3	9	18.81	0.0269
4	18	23.92	0.1576
5	27	30.92	0.2743
6	36	44.60	0.1540
7	45	50.23	0.2738
8	54	64.52	0.1547
9	63	72.96	0.1832
10	72	78.00	0.2940
11	81	87.64	0.2876
12	90	96.86	0.2919

Univariate Model ANOVA Diagnostics

		Standard		
Variable	R-Square	Deviation	F Value	Pr > F
dlwpi	0.4808	0.00914	4.01	0.0011
dliip	0.9561	0.01752	94.43	<.0001
dlm1	0.7564	0.02028	13.46	<.0001

Univariate Model White Noise Diagnostics

	Durbin	Normality		ARCH	
Variable	Watson	Chi-Square	Pr > ChiSq	F Value	Pr > F
dlwpi	2.04045	7.33	0.0256	0.92	0.3413
dliip	2.18189	1.19	0.5502	1.01	0.3208
dlm1	1.99207	0.83	0.6607	0.11	0.7423

Appendix 3: Percentage Root Mean Square Error (PRMSE) for WPI based on different combination of lambda and theta

		lambda									
		1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
theta	0.9	1.4796	1.4770	1.4736	1.4691	1.4633	1.4560	1.4474	***1.4400***	1.4446	1.4897
	0.8	1.4772	1.4741	1.4704	1.4656	1.4597	1.4525	1.4450	1.4407	1.4510	1.4990
	0.7	1.4739	1.4705	1.4666	1.4615	1.4555	1.4491	1.4435	1.4434	1.4603	1.5091
	0.6	1.4697	1.4660	1.4617	1.4566	1.4514	1.4465	1.4442	1.4497	1.4732	1.5195
	0.5	1.4640	1.4603	1.4561	1.4518	1.4481	1.4462	1.4489	1.4611	1.4900	1.5300
	0.4	1.4571	1.4540	1.4510	1.4488	1.4484	1.4515	1.4607	1.4799	1.5108	1.5399
	0.3	1.4511	1.4501	1.4503	1.4526	1.4580	1.4681	1.4844	1.5078	1.5345	1.5486
	0.2	1.4562	1.4607	1.4672	1.4765	1.4890	1.5051	1.5239	1.5434	1.5578	1.5555
	0.1	1.5099	1.5199	1.5307	1.5419	1.5532	1.5637	1.5725	1.5775	1.5756	1.5599

Appendix 4: BVAR(2)

Estimation Method	**Maximum Likelihood Estimation**
Prior Lambda	**0.3**
Prior Theta	**0.9**

Seasonal Constant Estimates				
Variable	Constant	Season 1	Season 2	Season 3
dlwpi	0.01506	-0.01268	-0.01328	-0.00170
dliip	0.04363	0.02134	0.00649	-0.13811
dlm1	0.00979	0.02926	0.06049	0.01558

Model Parameter Estimates

Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
dlwpi	CONST1	-0.01287	0.01307	-0.98	0.3295	1
	SD_1_1	0.01495	0.01824	0.82	0.4162	S_1t
	SD_1_2	0.01601	0.01596	1.00	0.3209	S_2t
	SD_1_3	0.01869	0.01541	1.21	0.2308	S_3t
	AR1_1_1	0.27226	0.13315	2.04	0.0463	dlwpi(t-1)
	AR1_1_2	-0.01495	0.10841	-0.14	0.8909	dliip(t-1)
	AR1_1_3	0.13876	0.08431	1.65	0.1062	dlm1(t-1)
	AR2_1_1	-0.31337	0.13668	-2.29	0.0262	dlwpi(t-2)
	AR2_1_2	0.08893	0.10384	0.86	0.3959	dliip(t-2)
	AR2_1_3	0.21975	0.08622	2.55	0.0140	dlm1(t-2)
dliip	CONST2	0.01950	0.01624	1.20	0.2357	1
	SD_2_1	0.00006	0.02267	0.00	0.9980	S_1t
	SD_2_2	0.03717	0.01984	1.87	0.0669	S_2t
	SD_2_3	-0.10122	0.01915	-5.29	0.0001	S_3t
	AR1_2_1	-0.53137	0.16548	-3.21	0.0023	dlwpi(t-1)
	AR1_2_2	0.21896	0.13473	1.63	0.1106	dliip(t-1)
	AR1_2_3	0.21129	0.10478	2.02	0.0492	dlm1(t-1)
	AR2_2_1	0.03900	0.16987	0.23	0.8194	dlwpi(t-2)
	AR2_2_2	-0.14044	0.12905	-1.09	0.2818	dliip(t-2)
	AR2_2_3	0.31351	0.10715	2.93	0.0052	dlm1(t-2)
dlm1	CONST3	-0.00311	0.02139	-0.15	0.8850	1
	SD_3_1	0.04310	0.02985	1.44	0.1552	S_1t
	SD_3_2	0.06106	0.02612	2.34	0.0235	S_2t
	SD_3_3	0.05923	0.02522	2.35	0.0229	S_3t
	AR1_3_1	-0.21334	0.21793	-0.98	0.3324	dlwpi(t-1)
	AR1_3_2	0.05611	0.17744	0.32	0.7532	dliip(t-1)
	AR1_3_3	-0.29889	0.13799	-2.17	0.0352	dlm1(t-1)
	AR2_3_1	-0.14122	0.22371	-0.63	0.5308	dlwpi(t-2)
	AR2_3_2	0.24353	0.16995	1.43	0.1582	dliip(t-2)
	AR2_3_3	0.17355	0.14112	1.23	0.2246	dlm1(t-2)

Schematic Representation of Cross Correlations of Residuals Variable

Lag	0	1	2	3	4	5	6	7	8	9	10	11	12
dlwpi	+..	...	...	...	...	...	...	...	...	...	...	...	...
dliip	.+.												
dlm1	..+												
+ is > 2*std error, - is < -2*std error, . is between													

Portmanteau Test for Cross Correlations of Residuals

Up to Lag	DF	Chi-Square	Pr > ChiSq
3	9	18.36	0.0312
4	18	22.65	0.2045
5	27	29.51	0.3366
6	36	43.71	0.1766
7	45	49.10	0.3123
8	54	62.21	0.2071
9	63	68.45	0.2976
10	72	73.81	0.4188
11	81	82.72	0.4259
12	90	91.71	0.4301

Univariate Model ANOVA Diagnostics

Variable	R-Square	Std. Deviation	F Value	Pr > F
dlwpi	0.4768	0.00818	3.95	0.0012
dliip	0.9542	0.01596	90.31	<.0001
dlm1	0.7499	0.01833	12.99	<.0001

Univariate Model White Noise Diagnostics

		Normality		ARCH	
Variable	Durbin Watson	Chi-Square	Pr > ChiSq	F Value	Pr > F
dlwpi	2.10694	6.85	0.0326	0.56	0.4564
dliip	1.99104	0.17	0.9202	0.84	0.3647
dlm1	2.02638	0.99	0.6106	0.14	0.7105

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