Causal Relationship between Saving, Investment and Economic Growth for India
– What does the Relation Imply?

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This study investigates the relationship between saving, investment and economic growth for India over the period 1950-51 to 2007-08. The literature on the role of saving in promoting economic growth generally points to saving led growth. However, few studies show evidence for growth driven saving and some suggest no relationship. In theory, saving may stimulate economic growth, economic growth may also induce saving. This paper adds to the literature by analysing the existence and nature of these causal relationships. The present analysis focuses on India, where saving rate has been the most pronounced. The co-integration analysis suggests that there is a long-run equilibrium relationship. The results of Granger causality test show that higher saving and investment lead to higher economic growth, but the reciprocal causality is not observed. Further, it is empirically evident that saving and investment led growth is coming from the household sector. It may be inferred from the results that India is not too close to the technological frontier and hence not catching up with the new technologies.

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Introduction

The relationship between saving, investment and economic growth has puzzled economists ever since economics became a scientific discipline. Generally, a portion of income is saved and put into investment. In a closed economy, the economy as a whole can save only as much as its income. The economy as a whole may reduce the consumption expenditure in relation to a given level of income and consequently increase its propensity to save. An exogeneous increase in the desire to save leads to an unchanged level of saving but at a lower level of income. If we define both saving and investment as the

difference between gross domestic product and consumption, it may tend to be interpreted in terms of cause-and-effect relationship.

The role of domestic saving and domestic investment in promoting economic growth has received considerable attention in India and also in many countries around the world. The central idea of Lewis's (1955) traditional theory was that an increase in saving would accelerate economic growth, while the early Harrod-Domar models specified investment as the key to promoting economic growth. On the other hand, the neoclassical Solow (1956) model argues that the increase in the saving rate boosts steady-state output by more than its direct impact on investment, because the induced rise in income raises saving, leading to a further rise in investment. Jappelli and Pagano (1994) claimed that saving contribute to higher investment and higher GDP growth in the short-run, whereas, the Carroll-Weil hypothesis (Carroll and Weil, 1994) states that it is economic growth that contributes to saving, not saving to growth.

The optimism about the Indian economy has been on an ascent in recent years. This has led to a resurgence of interest in the linkages among saving, investment and economic growth in India. Further, the recent empirical literature on saving made the interest towards the themes of capital accumulation, technological progress and economic growth - a shift away from the 1980s and the 1990s when discourse on macroeconomic issues was dominated by concerns with short term stabilisation and adjustment. Since the inception of economic planning in India, the emphasis has been on saving and investment as the primary instruments of economic growth and increase in national income. One of the objectives of economic plan (for *e.g.*, Eleventh five year plan) is to increase the production in the economy and thus economic growth. To increase the production, capital formation is considered as the crucial determinant; and capital formation has to be backed by the appropriate volume of saving. Increase in saving, use of the increased saving for increased capital formation, use of the increased capital formation for increasing saving, and use of the increased saving for a further increase in capital formation constituted the strategy behind economic growth. Though, classical growth models support the hypothesis of saving promoting economic growth, Carroll-Weil hypothesis contradicts with the argument.

In the Indian context, though empirical studies exist on the role of saving and investment in promoting economic growth, these provide only partial analysis. Moreover, some empirical studies support the classical growth theory, some studies agree with the Carroll-Weil hypothesis and some do not support

either of these. To illustrate, Sinha (1996) looked at the causality between the growth rates of gross domestic saving and economic growth, and found that there was no causality running in either direction. In a study, Mühleisen (1997) found significant causality from growth to saving but rejected causality from saving to growth for all forms of saving. In another study, Sinha and Sinha (2008) examined the relationships among growth rates of the GDP, household saving, public saving and corporate saving for the period 1950 to 2001 and found that economic growth produced higher saving in various forms and never the other way around. Verma (2007) employed the ARDL co-integration approach to determine the long run relationship of GDS, GDI and GDP for the period 1950-51 to 2003-04 and supported the Carroll-Weil hypothesis that saving does not cause growth, but growth causes saving.

It appears that there is no comprehensive study available on the analysis of the interdependence between saving and investment of the household, private corporate and public sectors with that of economic growth. Therefore, this article investigates the possibility of saving investment led growth and growth driven saving investment hypothesis, in detail, by testing for Granger causality between the logarithms of saving, nominal investment and nominal GDP in India. The paper is organised in four sections. Section 2 presents the behavior of saving, investment and national income in India over the past few decades. Section 3 discusses the data and econometric analysis along with the empirical results. Finally, concluding observations are presented in section 4.

Section II
Saving, Investment and National Income

Trends

Saving rate has steadily increased over time, from an extremely low base of 9.0 percent in 1950-51 to 37.7 percent in 2007-08 (Chart 1). A significant positive and robust relationship between growth rate and saving rate was observed during this period, as growth rate was also rising during this period. At the same time, investment rate has steadily increased, from a low base of 10.7 percent in 1950-51 to an all time high of 39.1 percent in 2007-08. Given that India had a closed capital account before 1991 which restricted capital mobility through administrative controls and outright prohibition, domestic saving and domestic investment in India were highly correlated (correlation coefficient is 0.99 percent for the entire period). It may be observed that the divergence between saving and investment is persistent until the liberalization

**Chart 1: Saving, Investment rates and nominal growth**
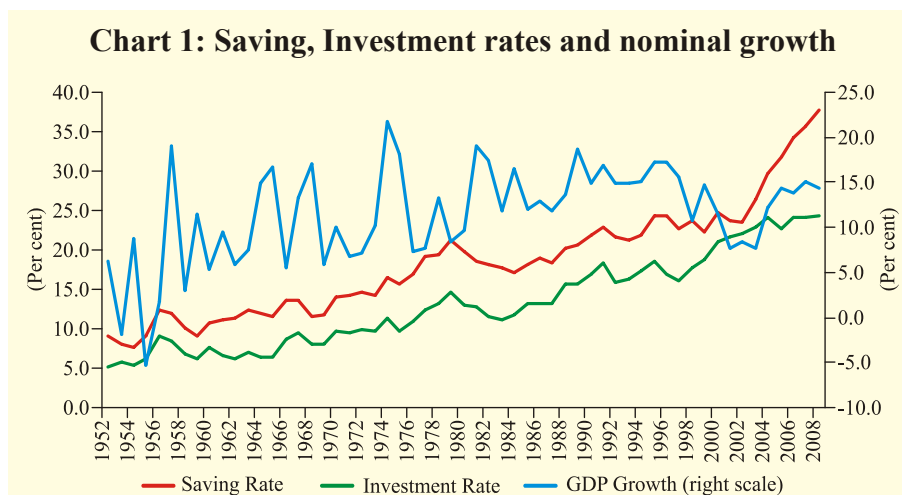
and was narrowed down after the 1991 balance of payments crisis and further narrowed down after the economy shifted to a flexible exchange rate regime in 1993. The correlation between saving and investment in the post reform period is more or less unchanged from the pre-reform period (correlation in the pre-reform period is 0.9973 and in post reform period is 0.9972), however the gap between them has narrowed.

As is evident from Chart 1, economic growth was largely led by investment demand, which is captured by the gross domestic fixed capital formation in national accounts. Though growing foreign investment, both direct and portfolio investment play a role, the rise in investment was largely financed domestically. From a low of 21.6 per cent in 1991-92, India's domestic saving rate jumped to a record high of 37.7 per cent in 2007-08. This fuelled investment, raising the demand for all types of investment related goods. This, in turn, had a multiplier effect on economic growth.

Composition

Domestic saving (Investment) of India is divided into two parts - Public Saving (Investment) and Private Saving (Investment). Private Saving (Investment) is further divided into two parts, those are Household Saving (Investment) and Corporate Saving (Investment).

While India's saving and investment rates have steadily increased over time, their composition has undergone a considerable change (Chart 2). The most noticeable trend is the growing divergence between the public and private saving. Public saving declined from its peak level of 4.9 per cent of GDP in

Chart 2: Composition of Saving
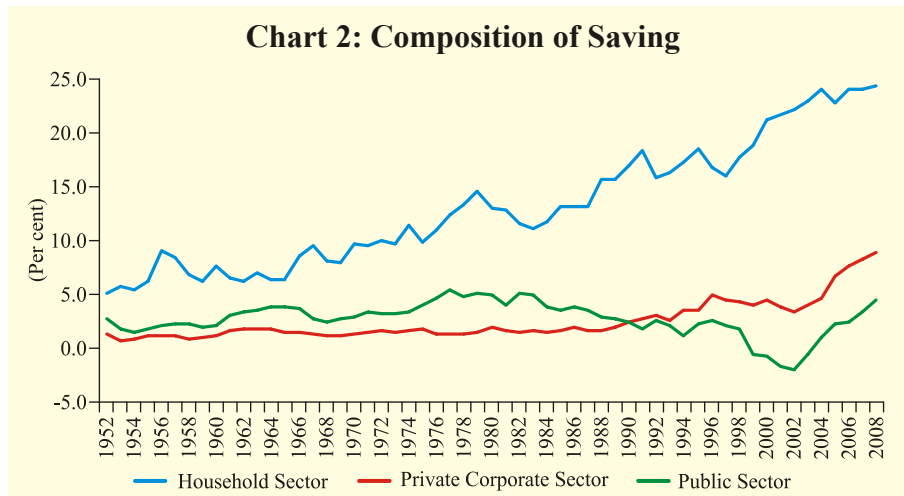
1976-77 to –2.2 per cent in 2001-02, from where it increased to 4.5 per cent in 2007-08. During the same period, saving rates of both the household and private corporate sectors have steadily increased, offsetting the decline in the public sector. The share of household saving in the total saving has increased from nearly 60 per cent in the early 1990s to a maximum of 94 per cent in 2001-02, after which it steadily declined to nearly 65 per cent in 2007-08. The private corporate sector, whose saving rate was stagnant till the late 1980s, has recently emerged as the sector with the fastest rising saving rate (1.8 per cent of GDP in 1987-88 to 8.8 per cent of GDP in 2007-08). The share of private corporate saving in total saving has increased from below 10 per cent in 1980s to more than 23 per cent in recent years.

Similar compositional changes have occurred in investment as well. Until late 1980s public investment rate was dominating and reached its peak of 12 per cent in 1986-87. Following the liberalisation in early 1990s, the role of public sector has gradually reduced in number of sectors, and its place has been taken over by the private sector. Hence, the private corporate investment has steadily increased offsetting the decline in the public sector investment. The share of public sector investment in total investment was stagnant at around 50 per cent till 1980s, and has declined to 23 per cent in 2007-08. On the other hand, the share of private corporate investment, which was little more than 20 per cent in 1980s, has steadily increased to 40 per cent in 2007-08. Household sector investment rate also increased from low base of 3.2 per cent in 1963-64 to 14.2 per cent in 2004-05 and it moderated thereafter. However, its share in total investment broadly remained the same.

Chart 3: Composition of Investment
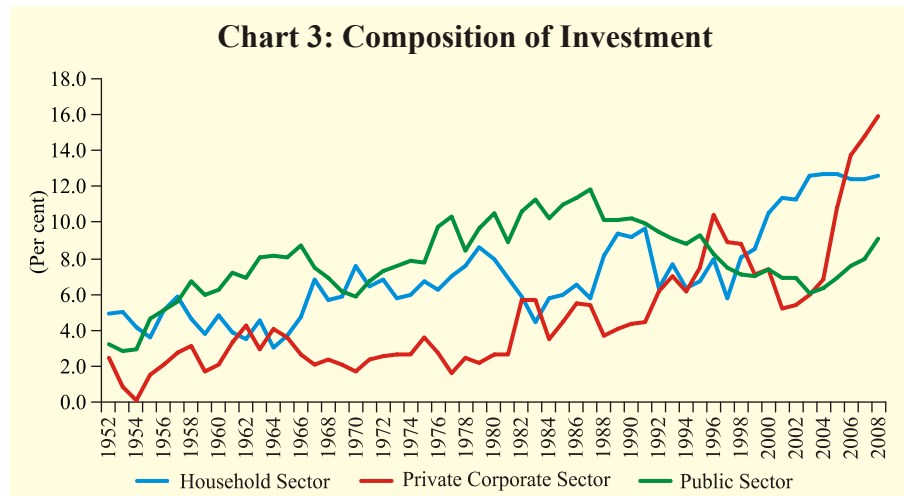
Section III
Econometric Analyses

Data

To understand the saving, investment led growth or growth driven saving and investment in India, we adopt Johansen methodology as given in Annex. The study uses the annual data to examine the causal relationships between domestic saving, investment and income for India. Annual time series data for gross domestic product (GDP), gross domestic saving (GDS), gross domestic investment (GDI), saving and investment of household sector, private corporate sector and public sector for the period 1950-51 to 2007-08 are collected from the National Accounts Statistics, published by the Ministry of Statistics and Programme Implementation, Government of India. All data are in terms of domestic currency and nominal prices.

Unit Root Test

One of the most important attributes of a time series variable is its order of integration. Hence, we first perform unit root tests in levels and first differences in order to determine the order of integration of the series. To test the order of integration, we employ the conventional augmented Dickey-Fuller (ADF) test (*Dickey and Fuller, 1979 and 1981*). ADF test examines the null hypothesis of a unit root against a stationary alternative. The results are presented in Table 1.

Table 1: Unit Root Test using Augmented Dickey Fuller Test

Variable	At level$_\mu$		At level$_\lambda$		At first difference$_\mu$		Concl-usion
	Optim-um Lag-length	ADF test statistic	Optim-um Lag-length	ADF test statistic	Optim-um Lag-length	ADF test statistic	
Gross Domestic Product (GDP)	0	3.47	1	-3.46	0	-5.34*	**I(1)**
Gross Domestic Saving (GDS)	0	2.14	0	-2.86	0	-6.45*	**I(1)**
Household Saving (HHS)	0	1.29	0	-3.22	0	-7.96*	**I(1)**
Private Corporate Saving (PCS)	0	1.36	0	-1.97	0	-8.31*	**I(1)**
Public Sector Saving (PBS)	2	0.68	0	-3.81**	-	-	**I(0)**
Private Sector Saving (PS)	0	1.83	0	-2.87	0	-7.10*	**I(1)**
Gross Domestic Investment (GDI)	0	1.29	0	-2.64	0	-7.84*	**I(1)**
Household Investment (HHI)	1	1.22	0	-3.41	0	-9.09*	**I(1)**
Private Corporate Investment (PCI)	8	0.65	0	-3.38	7	-4.01*	**I(1)**
Public Sector Investment (PBI)	0	-0.21	0	-2.53	0	-7.42*	**I(1)**
Private Sector Investment (PI)	2	2.13	0	-3.22	0	-7.41*	**I(1)**

Note: * and ** indicate statistical significance at 1% and 5% levels, respectively. The subscripts µ and λ indicate the models that allow for a drift term and a deterministic trend, respectively.

It is evident from the table that the calculated ADF statistics for level variables are less than the critical values in all cases, suggesting that the variables are not level stationary. Table 1 also shows that the ADF statistics for all the variables imply first-difference stationary, except for public sector saving (PBS). For further analysis, series whose order of integration is same as that of the GDP series are only retained for empirical analysis. Therefore, the series PBS has not been considered for further analysis.

Co-integration Test

Having established that all variables, except PBS, are integrated of same order, we proceed to test for presence of co-integration among the variables. We employ Johansen co-integration test. It may be noted here that we are interested to check for the presence of co-integrating relationship among the variables, however, number of co-integrating vectors is not of our interest. Accordingly, in Table 2, we present only the results of the null hypothesis that there does not exist co-integration against the alternative that there exists co-integration.

Starting with the null hypothesis that co-integration (r=0) does not exist among the variables, the trace statistic is well above the 95 per cent critical

Table 2: Empirical Results of the Co-integration Test based on Johansen-Juselius method

H_0: There does not exist co-integration

Variables in the system	Trace statistic	Maximum Eigen value statistic	Conclusion
GDP and GDS	24.33 *	18.03 *	Co-integrated
GDP and GDI	34.06 *	29.55 *	Co-integrated
GDP, GDS and GDI	43.46 *	30.74 *	Co-integrated
GDP and PS	29.94 *	22.48 *	Co-integrated
GDP and PI	27.19 *	21.54 *	Co-integrated
GDP, PS and PI	50.01 *	24.33 *	Co-integrated
GDP and HHS	23.95 *	17.08 *	Co-integrated
GDP and HHI	19.75 *	16.36 *	Co-integrated
GDP, HHS and HHI	39.33 *	21.71 *	Co-integrated
GDP and PCS	15.22	10.47	Not co-integrated
GDP and PCI	39.59 *	34.93 *	Co-integrated
GDP, PCS and PCI	53.79 *	41.73 *	Co-integrated
GDP and PBI	32.69 *	32.63 *	Co-integrated

Note: * indicate statistical significance at 5% levels. The critical values of Trace test and Maximum Eigen value test at the 5% significance levels are 15.4947 and 14.2646, respectively.

value for all the series except private corporate sector saving (PCS). Hence, it rejects the null hypothesis of no co-integration in favor of existence of co-integration for all the series except PCS. Turning to the maximum eigen value test, the null hypothesis that there does not exist co-integration is rejected at 5 per cent level of significance in favor of the specific alternative that there is at least one co-integrating vector for all series except PCS. Thus, both the trace and maximum eigen value test statistics suggest that there exist co-integration relationship among all series with GDP except PCS. Hence, we use Vector Error Correction (VEC) Model for all other series and Vector Auto Regression (VAR) Model for PCS to test for causality.

Since GDP is co-integrated with GDS and GDI individually as well as collectively for the Indian economy, one can infer that there is a long-run equilibrium relationship between the two series and existence of causality in at least one direction. Private sector's saving and investment is also co-integrated with the national income suggesting the existence of long-run equilibrium relationship between national income and saving and investment of private sector. It is evident from the empirical results that there does not exist co-integrating relationship between national income and private corporate sector saving. It may be noted that the existence of co-integration relationship between national income and saving and investment of private sector is mainly because of the households sector rather than the private corporate sector.

Granger Causality

Given the results of the co-integration tests, one has to estimate the VECM/ VAR to determine the direction of causality between income, saving and investment. If co-integration exists, the Granger-Causality test is performed under the vector error correction methodology. Otherwise, as in the case of saving of private corporate sector and gross domestic product, the standard Granger-Causality test is performed under VAR framework. The results of the causality tests under the VECM/VAR framework are shown in Table 3.

The bivariate Granger causality tests performed under VECM framework between saving and income and between investment and income, show that there is uni-directional causality between gross domestic saving and national income and also between gross domestic investment and national income. In line with the existing literature, it is evident from the empirical results that the causality is running from saving to income rather than income to saving. It is further evident that investment leads to higher income, whereas, income does not lead to higher investment. Under three variable VECM framework, it is empirically found that saving and investment collectively lead to higher income in India. However, income does not lead to higher saving and investment.

Further, it is evident that private sector saving causes higher growth and vice-versa, whereas, private sector investment alone may not boost the economic growth. Moreover, private sector surplus both in the form of saving and investment would boost economic growth. The causation of growth from household sector and private corporate sector is further investigated separately. It is empirically found that household saving is endogenous to growth, but household investment is not endogenous to growth. On the other hand, household sector saving and investment collectively are endogenous to growth.

Bivariate granger causality test under VAR framework is employed for private corporate sector saving and national income and it is found that national income leads to private corporate sector saving but not the vice-versa. In the case of private corporate sector investment and national income, the test is performed under the VECM framework. It is found that private corporate sector investment leads to higher growth and growth causes higher investment in the private corporate sector. Further, it is found that saving and investment of private corporate sector are endogenous to growth collectively. Moreover, higher investment in the public sector improves economic growth, whereas, higher growth does not necessarily foster higher investment in the public sector.

Table 3: Causality tests based on VECM/VAR: F statistic

Null Hypothesis	F-Statistic	Result
Entire economy		
Gross domestic saving does not granger cause Gross domestic product	19.05	Reject
Gross domestic product does not granger cause Gross domestic saving	1.39	Do not Reject
Gross domestic investment does not granger cause Gross domestic product	18.88	Reject
Gross domestic product does not granger cause Gross domestic investment	2.53	Do not Reject
Gross domestic saving and investment does not granger cause GDP	21.33	Reject
GDP does not granger cause Gross domestic saving and investment	4.95	Do not Reject
Private sector		
Private sector saving does not granger cause Gross domestic product	9.94	Reject
Gross domestic product does not granger cause Private sector saving	7.07	Reject
Private sector investment does not granger cause Gross domestic product	1.28	Do not Reject
Gross domestic product does not granger cause Private sector investment	15.49	Reject
Private sector saving and investment does not granger cause Gross domestic product	10.29	Reject
Gross domestic product does not granger cause Private sector saving and investment	17.97	Reject
Household sector		
Household sector saving does not granger cause Gross domestic product	9.92	Reject
Gross domestic product does not granger cause Household sector saving	7.89	Reject
Household sector investment does not granger cause Gross domestic product	3.99	Do not Reject
Gross domestic product does not granger cause Household sector investment	17.32	Reject
Household sector saving and investment does not granger cause Gross domestic product	26.11	Reject
Gross domestic product does not granger cause Household sector saving and investment	8.80	Reject
Private corporate sector		
Private corporate sector saving does not granger cause GDP	1.78	Do not Reject
GDP does not granger cause Private corporate sector saving	7.50	Reject
Private corporate sector investment does not granger cause GDP	6.06	Reject
Gross domestic product does not granger cause Private corporate sector investment	19.78	Reject
Private corporate sector saving and investment does not granger cause GDP	8.60	Reject
GDP does not granger cause Private corporate sector saving and investment	9.00	Reject
Public sector		
Public sector investment does not granger cause Gross domestic product	22.03	Reject
Gross domestic product does not granger cause Public sector investment	1.07	Do not Reject

Discussion of the empirical results

All long-run growth theories imply that an economy can grow faster by investing more. An economy with open capital markets, *viz.*, India, may not need higher domestic savings to grow faster as investment can be financed by foreign sources. However, the empirical results suggest that higher domestic saving would boost economic growth. The positive correlation between saving and growth appears rather puzzling from the point of view of standard growth theory. Some researchers, for example Carroll-Weil (1994) have sought to explain the correlation as reflecting an effect of growth on saving. But this interpretation runs counter to mainstream economic theory in which the representative individual's consumption-Euler equation implies that growth should have a negative effect on saving. India being an open economy with domestic and foreign investors, domestic saving need not be endogenous to growth.

Growth in emerging economy results mainly from innovations that allow domestic sectors to catch up with the current frontier technology. But catching up with the frontier in any sector requires the cooperation of a foreign investor who is familiar with the frontier technology and a domestic entrepreneur who is familiar with the local conditions to which the technology must be adapted.

When domestic saving causes economic growth, as is empirically found for India, the question arises as to how far the country is from the technological frontier. Particularly, focus will be on the interaction between saving and the country's distance from the technological frontier. *Aghion et al (2006)* argues that saving affects growth positively in those countries that are not too close to the technological frontier, but does not affect it at all in countries that are close to the frontier. The reason explained is that, higher saving in an emerging economy increases the number of projects that can be co-financed by the local entrepreneur on terms that mitigate agency problems enough to make it worthwhile for a foreign investor to participate. However, in countries sufficiently close to the frontier, the local firms are more likely to be familiar themselves with the frontier technology, and therefore do not need to attract foreign investment in order to undertake an innovation project. In such a case, every ex ante profitable innovation project will be undertaken regardless of the level of domestic saving because there is no need for co-financing when there is just one agent participating in a project.

Section IV
Summary and Conclusions

The study examines the direction of the relationship between saving, investment and economic growth in India at both aggregate level and sectoral level for the period 1950-51 to 2007-08 by using Granger causality test. It is empirically evident that the direction of causality is from saving and investment to economic growth collectively as well as individually and there is no causality from economic growth to saving and (or) investment.

The empirical results suggest that there exists reciprocal causality from saving and investment of the private sector to economic growth. This reciprocal causality emanates from the household sector, where saving and investment led growth and growth driven saving and investment was observed. It is empirically evident that private corporate sector saving does not lead to economic growth, however, saving and investment of the sector collectively lead to economic growth and vice-versa.

Saving led growth in emerging market economies implies that the economy is not catching up with the technology frontier and hence growth is not driven by the innovations that are taking place worldwide. The results indicate that though the Indian economy is opened to foreign investments, growth is still driven by the domestic saving. Furthermore, local firms may not be absorbing the technology which comes through the foreign investment in order to undertake more profitable innovation projects.

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Annex
Econometric methodology used

Granger (1969, 1980) is well known for his Granger causality test. The concept of Granger causality, by which we actually understand precedence, is based on the idea that a cause cannot come after its effect. More precisely, variable X is said to Granger cause another variable Y, if the current value of Y is conditional on the past values of X and thus the history of X is likely help to predict Y.

The Granger causality method regresses variable Y on its own lagged values (Y_{t-i}) and the lagged values of another variable X (X_{t-i}). If the coefficients of lagged values of X are significant, then X Granger causes Y. Similarly, to substantiate the reverse possibility, one regresses X on its own lagged values and lagged values of Y. Y Granger causes X if the coefficients of the lagged values of Y are significant. In summary, Granger causality tests can be placed in one of four categories: No causality, Y causes X only, X causes Y only, and a bi-directional causality, *i.e.*, Y causes X and X causes Y simultaneously.

Steps involved in implementing the Granger causality test:

1. Test for the stationarity of the data using Augumented Dickey Fuller (ADF) test.

2. If found non-stationary, difference the data and conduct the ADF test again on the differenced data.

3. Exclude the variables, whose order of integration is not the same as order of integration of GDP.

4. Test for the presence of co-integration using the same order of integrated variables.

5. Based on co-integration results, use VAR or VEC to test causality.

The first step in the causality testing procedure is to determine whether the data contains unit roots indicating the data is non-stationary. To formally test for the presence of unit root, the conventional augmented Dickey- Fuller (ADF) test (*Dickey and Fuller, 1979 and 1981*) is employed. The following regression equations are used to test for the presence of unit root.

$$\Delta Y_t = \mu + (\rho - 1)Y_{t-1} + \sum_{i=1}^{p} \theta_i \Delta Y_{t-i} + \varepsilon_t \tag{1}$$

$$\Delta Y_t = \mu + \beta t + (\rho - 1)Y_{t-1} + \sum_{i=1}^{p} \theta_i \Delta Y_{t-i} + \varepsilon_t \tag{2}$$

where Δ is the differencing operator

Y_t is logGDP (or logorthim of GDS, HHS, PCS, PBS, GDI, HHI, PCI, PBI) at time t

p is the maximum lag length

ε is the stationary random error.

Equation (1) is a test for random walk with drift term (intercept), whereas, equation (2) tests for random walk with drift term and linear trend. Basically, one would use the most general case and estimate a regression with both the drift term and linear time trend, and step-by-step estimate the restricted equations, if the test fails to reject the null hypothesis of unit root present in the general case. The null hypothesis is that unit root present in the series (*i.e.*, $\rho=1$ or $\rho-1=0$). The series is said to be stationary or do not have unit root, if $1-\rho$ is negative and statistically significant.

Once we have the results of unit roots, the next step is to determine whether there exists co-integration, using the same order of integrated variables. To test for co-integration, the *Johansen and Juselius (1990)* procedure is used, which leads to two test statistics, trace test and maximum eigenvalue test, for co-integration[1]. The two test statistics, λ_{trace} and λ_{max} are used to estimate the co-integration rank r, *i.e.*, the number of independent co-integrating vectors.

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} \ln(1 - \lambda_i) \tag{3}$$

$$\lambda_{max}(r, r+1) = -T \ln(1 - \lambda_{r+1}) \tag{4}$$

where $\lambda_{r+1}, \ldots, \lambda_n$ are the estimated (n-r) smallest eigenvalues

T is the number of usable observations.

The distribution of statistics is subject to whether a constant or a drift term is included in the co-integrating vector and the number of non-stationary components under the null hypothesis.

If the rank r is zero, the variables are not co-integrated and hence the vector auto regression (VAR) method would be used to investigate causality. On the other hand, if the variables are co-integrated, the vector error correction (VEC) method is used to test for causality[2].

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[1] Trace test, tests the hypothesis that there are at most r co-integrating vectors, whereas, maximum eigenvalue test, tests the hypothesis that there are r+1 co-integrating vectors versus the hypothesis that there are r co-integrating vectors.

[2] See Toda and Phillips (1994) for a detailed discussion.