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Estimating Value at Risk (VaR) using Filtered Historical Simulation in the Indian capital market

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The paper estimates Value at Risk (VaR) of the daily return of Indian capital market (SENSEX/NIFTY) using Filtered Historical Simulation (FHS). It uses GARCH framework to model the volatility clustering on returns and examines the usefulness of considering lag values of return (on S&P 500, INR-EURO INR-USD exchange rate, gold price) as proxies of global financial condition in the specification of the mean equation. In general, VaR is calculated using (i) Historical Simulation approach which imposes no structure on the distribution of returns except stationarity and (ii) Monte Carlo simulation approach which assumes parametric models for variance and subsequently a large sample of random members is drawn from this specific distribution to calculate the VaR. FHS approach attempts to combine the model-based approach with the model-free approaches. The VaR is estimated based on two approaches. In the first approach, the mean equation of daily return in Indian capital market is captured by its own lag and daily return of S&P-500, INR-EURO, INR-USD exchange rate and gold price; while volatility is modeled by GARCH model and finally the VaR is estimated through FHS. In the second approach, the mean equation is being captured by ARMA model, while volatility is modeled by GARCH model and finally the VaR is estimated through FHS. It is observed that VaR estimated using (a) GARCH with suitable mean specification, outperforms method (b) based on ARMA-GARCH.

JEL classification : G1, C52
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Introduction

Globalisation and financial sector reforms in India led to a greater integration of Indian stock market with the advanced economies and also to the exchange rate movements. In the early 1960s, Eugene Fama

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developed efficient market hypothesis (EMH) which describes financial market as informational efficient. In an efficient market, actual price of a security will be a good estimate of its intrinsic value. Fama illustrated three forms of market efficiency, *i.e.*, weak form, semi-strong form and strong form of market efficiency based on the availability of information. According to weak form of EMH, all past market prices and data are fully reflected in securities prices. In other words, technical analysis cannot be used to predict and beat a market. The semi-strong form of EMH assumes that all publicly available information is fully reflected in securities prices which essentially implies that fundamental analysis is of no use. Strong form of EMH assumes that market reflects even hidden/inside information. In other words, according to strong form of EMH, even insider/hidden information is of no use. The weak form of market efficiency hypothesis has been tested by Fama (1970) for U.S., Dryden (1970) for U.K., Conrad and Juttner (1973) for Germany, Jennergren and Korsvold (1975) for Norway and Sweden, Lawrence (1986) for Malaysia and Singapore, Andersen and Bollerslev (1997) for European markets. These studies provided indecisive results. The developed markets were found to be weak form efficient. On the other hand, evidence from emerging markets indicated rejection of the weak form market efficiency hypothesis. Therefore, question arises whether the returns in these emerging markets are predictable. Apart from the form of efficiency, it is the volatility prevailing in the market which influences the return to a large extent. Volatility, which refers to the degree of unpredictable change over time and can be measured by the standard deviation of a sample, often used to quantify the risk of the instrument of portfolio over that time period. Equity return volatility may be defined as the standard deviation of daily equity returns around the mean value of the equity return and the stock market volatility is the return volatility of the aggregate market portfolio. Engle (1982) introduced the concept of Autoregressive Conditional Heteroscedasticity (ARCH) which became a very powerful tool in the modelling of high frequency financial data. ARCH models allow the conditional variances to change through time as functions of past errors. Bollerslev (1986) made significant improvement on ARCH and introduced the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) process. Further, many more variations were introduced such as Integrated GARCH (IGARCH) by Engle and Bollerslev (1994) and the exponential GARCH (EGARCH) by Nelson (1991), where different re-specification of variance equation was studied.

In financial risk management, VaR is widely used as the risk measure and is defined as the maximum potential loss that would be incurred at a given probability p for a financial instrument or portfolio during a given period of time. In general, VaR is calculated either based on Historical Simulation (HS) approach, which imposes virtually no structure on the distribution of returns except stationarity, or using Monte Carlo simulation (MCS) approach which assumes parametric models for variance and subsequently large sample of random numbers is drawn from this specific distribution to calculate the desired risk measure. Filtered Historical Simulation (FHS) approach attempts to combine the best of the model-based with the best of the model-free approaches in a very intuitive fashion.

There have been some significant empirical studies on stock return volatility in emerging markets like India in recent years. However, there is hardly any study which estimated VaR following Filtered Historical Simulation approach using GARCH model with suitable mean specification, in the context of Indian capital market. Pattanaik and Chatterjee (2000) used ARCH/GARCH models to model the volatility in Indian financial market. Agarwal and Du (2005) using BSE 200 data found that the Indian stock market is integrated with the matured markets of the World. Raj and Dhal (2008) investigated the financial integration of India's stock market with that of global and major regional markets. They used six stock price indices, *i.e.*, the 200-scrip index of BSE to represent domestic market, stock price indices of Singapore and Hong Kong to represent the regional markets and three stock price indices of U.S., U.K. and Japan to represent the global markets. Based on daily as well as weekly data covering end-March 2003 to end-January 2008, they found that Indian market's dependence on global markets, such as U.S. and U.K., was substantially higher than on regional markets such as Singapore and Hong Kong, while Japanese stock market had weak influence on Indian market.

The paper examines the financial integration of Indian capital market (BSE-SENSEX and NSE-NIFTY) with other global indicators and its own volatility using daily returns covering the period January 2003 to December 2009. The paper specifies a GARCH framework to model the phenomenon of volatility clustering on returns and examines the usefulness of considering lag values of returns (on S&P 500, INR-EURO INR-USD exchange rate, gold price) as proxies to global financial conditions in the specification of the mean equation. The paper also estimates VaR of return in the Indian capital market based on two composite methods, *i.e.*, (a) using univariate GARCH model where in the mean equation we have used lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, Gold price) and following the filtered historical simulation (FHS) approach (b) using ARMA for mean equation, GARCH for volatility and FHS for VaR estimation, *i.e.*, ARMA-GARCH-FHS methods; and finally compares the performance of both the VaR estimates.

The rest of the paper is organised as follows. Section II describes the portfolio model using GARCH specifications, section III describes estimate of VaR based on HS, MCS and FHS. Section IV discusses the data and focuses on VaR calculation and summarising the results. Finally, section V concludes.

Section II
The Portfolio Model

In the financial literature, it is well documented that variance of asset returns, in general, changes over time and GARCH models are popular choice to model these changing variances. Let r_t; $t = 1, \ldots, T$, represents the continuously compounded rate of returns of a stock price index (for holding the portfolio for one day) at time t. If p_t is the stock price index then $r_t = l_n(p_t) - l_n(p_{t-1})$, where '$l_n$' is the natural logarithm. The model can be written as:

$$r_{t+1} = c + \phi_1 r_t + \phi_2 r_{t-1} + .. + \phi_k r_{t+1-k} + \psi_1 x_{1,t+1} + \psi_2 x_{2,t+1} + .. + \psi_s x_{s,t+1} + \sigma_{t+1}\eta_{t+1} ;$$
$$\sigma^2_{t+1} = \omega + \alpha \text{Resid}^2_t + \beta\sigma^2_t ; \; t=1,2\ldots T \qquad \textbf{(1)}$$

where $\text{Resid}_t = (r_t - c - \Sigma\phi_i r_{t-I} - \Sigma\psi_j x_{j,t})$; innovation $\{\eta_t\}$ is white noise process, with zero mean and unit variance and $\alpha+\beta < 1$, $X_1..X_s$ are the external factors influencing r_t.

Section III
Value at Risk

Value at Risk is being widely used as measure of market risk of an asset or of a portfolio. The Parametric VaR model imposes a strong theoretical assumption on the underlying properties of data; frequently Normal Distribution is assumed because it is well understood and can be defined using the first two moments (mean and standard deviation). Other probability distributions may be used, but at a higher computational cost. However, empirical evidence indicates that asset price changes, in particular the daily price changes, often do not follow normal distribution. In the presence of excess kurtosis, failure rate increases when the VaR is estimated by the Gaussian distribution. The 100α% one day ahead VaR ($\lambda_{\alpha,t}$) is defined as $P[r_t <= \lambda_{\alpha,t} \mid r_{t-1}] = \alpha$. In general, VaR techniques are based on non-parametric, parametric or mixture of parametric and non-parametric statistical methods. The family of Historical Simulation (HS) models is a non-parametric approach. The FHS as developed by Barone-Adesi *et al* (1998) and Barone-Adesi *et al* (1999, 2000) is a mixture of parametric and non-parametric approach.

Historical Simulation

Apart from stationarity of the returns, HS method does not require any statistical assumption in particular to the volatility. In HS method we consider the availability of a past sequence of daily portfolio returns for m days; r_t t=1,2…m. The HS technique simply assumes that the distribution of tomorrow's portfolio returns, r_{t+1}, is well approximated by the empirical distribution of the past *m* observations-that is, $\{r_{t+1-\tau}\}_{\tau=1..m}$. In other words, the distribution of r_{t+1} is captured by the histogram of $\{r_{t+1-\tau}\}_{\tau=1..m}$. Thus, we simply arrange the returns in $\{r_{t+1-\tau}\}_{\tau=1..m}$ in ascending order and choose the VaR^p_{t+1} to be a number such that only 100*p*% of the observations are smaller than the VaR^p_{t+1}.

Monte Carlo Simulation (MCS)

MCS can be explained better through an example. Let us consider GARCH(1,1) model as defined in equation (1), *i.e.:*

$r_{t+1}=c+\phi_1 r_t+\phi_2 r_{t-1}+..+\phi_k r_{t+1-k}+\psi_1 x_{1,t+1}+\psi x_{2,t+1}+..+\psi_s x_{s,t+1}+\sigma_{t+1}\eta_{t+1}$;
$t=1,2\ldots T$ $\sigma^2_{t+1}=\omega+\alpha Resid^2_t+\beta\sigma^2_t$

where $Resid_t=(r_t-c-\Sigma\phi_i r_{t-I}-\Sigma\psi_j x_{j,t})$; innovation $\{\eta_t\}$ is white noise process, with zero mean and unit variance and $\alpha+\beta<1$. Although in the case of daily asset returns, generally, η_t does not follow normal distribution but using other probability distributions is computionally very costly, let us assume η_t follows Normal Distribution N(0,1).

Based on the above specified GARCH model, at the end of day 't' we can calculate the variance of day 't+1', *i.e.*, $\sigma^{\wedge 2}_{t+1.}$

Let $\{\eta^{\wedge}_{i,1}$; i=1,2…L) be a set of large number of random numbers drawn from the standard Normal Distribution N(0,1). From these random numbers $\{\eta_{i,1}$; i=1,2…L}, we can calculate a set of hypothetical returns for day 't+1' as

$r^{\wedge}_{i,t+1}=c+\Sigma\phi_i r_{t+1-i}+\Sigma\psi_j x_{t+1-j}+\sigma^{\wedge}_{t+1}\eta_{i,1}{}^{\wedge}$; i=1,2…L

$Resid_{i,t+1}=(r^{\wedge}_{i,t+1}-c-\Sigma\phi_i r_{t+1-i}-\Sigma\psi_j x_{t+1-j})$

Given these hypothetical estimated returns ($r^{\wedge}_{i,t+1}$) for day 't+1', we can compute the hypothetical variances for the 't+2' day as :

$\sigma^{\wedge 2}_{t+2}=\omega+\alpha Resid^2_{t+1}+\beta\sigma^{\wedge 2}_{t+1}$

Similarly, to estimate the hypothetical return ($r^{\wedge}_{i,t+2}$) on day t+2, a large number of pseudo random numbers is drawn again from the *N(*0*,* 1*)* distribution, *i.e.,* $\{\eta_{i,2}$; i=1,2…L}

$r^{\wedge}_{i,t+2}=c+\Sigma\phi_i r_{t+2i}+\Sigma\psi_j x_{t+2j}+\sigma^{\wedge}_{t+2}\eta_{i,2}{}^{\wedge}$; i=1,2…L

$Resid_{i,t+2}=(r^{\wedge}_{i,t+2}-c-\Sigma\phi_i r_{t+2-i}-\Sigma\psi_j x_{t+2-j})$

and variance is now updated by

$\sigma^{\wedge 2}_{t+3}=\omega+\alpha Resid^2_{t+2}+\beta\sigma^{\wedge 2}_{t+2}$

Similarly, we can get the hypothetical return of 't+k' day

$r^{\wedge}_{i,t+k}=c+\Sigma\phi_{i*}r_{i},_{t+k-1}+\Sigma\psi_{j*}x_{j},_{t+k-1}+\sigma^{\wedge}_{t+k-1*}\eta_{i,k}{}^{\wedge}$; i=1,2…L

Therefore, hypothetical Kth return can be written as:

$\hat{r}_{i,t+1:t+k} = \Sigma_k \hat{r}_{i,t+k}$; i=1,2…L

If we collect these L hypothetical *K*-day returns in a set $\{\hat{r}_{i,t+1:t+k}$; i=1,2…L}, then the *K*-day VaR can be calculated as the 100p percentile, *i.e.:*

$VaR^{p}_{t+1:t+k} = -$ *Percentile* $[\{\hat{r}_{i,t+1:t+k}$;i=1,2…L},100p}]

Filtered Historical Simulation (FHS)

As we have discussed that non-parametric approach such as HS does not assume any statistical distribution of returns, whereas parametric approach such as the Monte Carlo simulation (MCS) takes the opposite view and assumes parametric models for variance, correlation (if a disaggregate model is estimated), and the distribution of standardised returns. Random numbers are then drawn from this distribution to calculate the VaR. Both of these extremes in the model-free/model-based spectrum have pros and cons. MCS is good if the assumed distribution is fairly accurate in description of reality. HS is sensible as the observed data may capture features of the returns distribution that are not captured by any standard parametric model. The FHS approach, on the other hand, attempts to combine the best of the MCS with the best of the HS.

Let us assume that we have estimated a GARCH-type model of our portfolio variance given in equation (1). Although we may be comfortable with our variance model (σ), we may not be comfortable in making a specific distributional assumption about the (η), such as a Normal or a *t* distribution. Instead of that, we might like the past returns data (r_t) to determine the distribution directly without making further assumptions.

Given a sequence of past returns and estimated GARCH volatility, $\{r_{t+1-\tau}, \hat{\sigma}_{t+1-\tau};\ \tau = 1,2\ldots m\}$ calculated past standardised returns are given by $\hat{\eta}_{t+1-\tau} = (r_{t+1-\tau} - E(r_{t+1-\tau}))/\hat{\sigma}_{t+1-\tau}$; τ =1,2…m

Instead of drawing random $\hat{\eta}$s from a specific probability distribution as it is done in MCS, in FHS method samples are drawn with replacement from $\{\hat{\eta}_{t+1-\tau};\ \tau = 1,2\ldots m\}$.Thereafter, we can get the hypothetical return of 't+k' day as:

$\hat{r}_{i,t+k} = c + \Sigma\phi_{i*} r_{i,t+k-1} + \Sigma\psi_{j*} x_{j,t+k-1} + \hat{\sigma}_{t+k-1*} \hat{\eta}_{i,k}$; i=1,2…L.

Therefore, hypothetical *K*-day return can be written as :

$\hat{r}_{i,t+1:t+k} = \Sigma_k \hat{r}_{i,t+k}$; i=1,2…L

The *K*-day VaR can be calculated based on L estimated k-day returns $\{\hat{r}_{i,t+1:t+k}\}$ as the 100p percentile, *i.e.,*

$VaR^p_{t+1:t+k} = -\ Percentile\ [\{\hat{r}_{i,t+1:t+k}; i=1,2\ldots L\}, 100p]$

Section IV
VaR Model Selection: Statistical Tests

Lopez (1998, 1999) formalised the use of loss functions as a means of evaluating VaR models and risk managers prefer the VaR model which maximises the utility function (minimises loss). Therefore, using utility functions in the evaluation of alternative VaR estimators is more effective than other nonparametric test such as Christoffersen's (1998) "conditional coverage" test. Lopez (1998,1999) proposed three loss functions, *viz.* the binomial loss function, the magnitude loss function and the zone loss function. Sharma, Thomas and Shah (2002) used a regulatory loss function to reflect the regulatory loss function (RLF), and a firm's loss function (FLF) which reflects the utility function of a firm. The regulatory loss function linked to the objectives of the financial regulator and the firm's loss function primarily focused in measuring the opportunity cost of firm's capital. Let r_t be the change in the value of a portfolio over a certain horizon and v_t is the VaR estimate at (1-p) level of significance.

Regulatory Loss Function (RLF)

It penalises failure differently from the binomial loss function, and pays attention to the magnitude of the failure.

$$l_t = \begin{cases} (rt - vt)^2 & if \quad rt < vt \\ 0 & otherwise \end{cases}$$

Firm's Loss Function (FLF)

There is a conflict between the goal of safety and goal of profit maximisation for an organisation which uses VaR for internal risk management. There is an opportunity cost of capital for the firm which uses a particular VaR model which specifies a relatively high value of VaR as compared to other VaR model. The FLF is defined as :

$$l_t = \begin{cases} (rt - vt)^2 & if \quad rt < vt \\ -\alpha v_t & otherwise \end{cases}$$

Where α measures the opportunity cost of capital.

Let $z_t = l_{it} - l_{jt}$, where l_{it} and l_{jt} are the values of a particular loss function generated by model i and model j respectively, for the day t. If θ is the median of distribution of z_t then the superiority of model *i* over model *j* with respect to a certain loss function can be tested by performing one-sided sign test.

$H_0 = \{\theta = 0\}$

$H_1 = \{\theta < 0\}$

Let $\psi_t = \begin{cases} 1 & if \quad z_t \geq 0 \\ 0 & otherwise \end{cases}$ and $Sij = \sum_{t=1}^{T} \psi_t$

The test statistics is

$$S_{ij}^{a} = \frac{S_{ij} - 0.5T}{\sqrt{.25T}} \sim \text{N(0,1) asymptotically.}$$

If $S_{ij}^{a} < -1.66$, H_0 is rejected at 5 percent level of significance, which would imply that model i is significantly better than model j.

The Diebold-Mariano test (1995) aims to test the null hypothesis of equality of expected forecast accuracy against the alternative of different forecasting ability across models. Let $\{y_t\}$ denote the series to be forecast and let $y^1_{t+h|t}$ and $y^2_{t+h|t}$ denote two competing forecasts of y_{t+h}. The forecast errors from the two models are :

$\varepsilon^1_{t+h|t} = y_{t+h} - y^1_{t+h|t}$

$\varepsilon^2 t+h|t = y_{t+h} - y^2_{t+h|t}$

Some common loss functions are :

Squared error loss: $L(\varepsilon^{i}_{t+h|t}) = (\varepsilon^{i}_{t+h|t})^2$

Absolute error loss: $L(\varepsilon^{i}_{t+h|t}) = |\varepsilon^{i}_{t+h|t}|$

The Diebold-Mariano test is based on the loss differential, *i.e.,*

$d_t = L(\varepsilon^{1}_{t+h|t}) - L(\varepsilon^{2}_{t+h|t})$

The null of equal predictive accuracy is then :

$H_0 : E[d_t] = 0$

The Diebold-Mariano test statistic is

$$S = \frac{\bar{d}}{\sqrt{(asymp\_variance(\bar{d})}} = \frac{\bar{d}}{\sqrt{(L\hat{RV}_{\bar{d}} / T)}}$$

where $\bar{d} = \frac{1}{T_0} \sum_{t=t_0}^{T} d_t$

$$LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\alpha} \gamma_j = \mathrm{cov}(d_t, d_{t-j})$$

$L\hat{RV}_{\bar{d}}$ is a consistent estimate of the asymptotic (long-run) variance of $\sqrt{T}\ \bar{d}$.

Diebold and Mariano (1995) show that under the null of equal predictive accuracy S ~ N(0, 1) asymptotically.

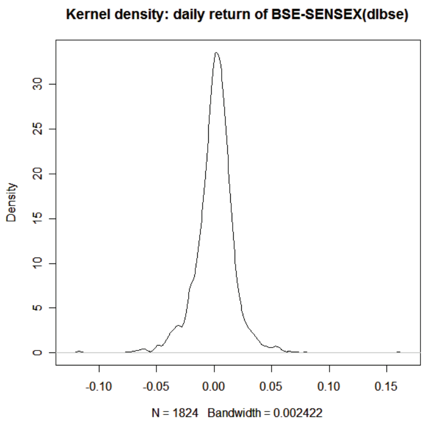
Section V
Empirical Results

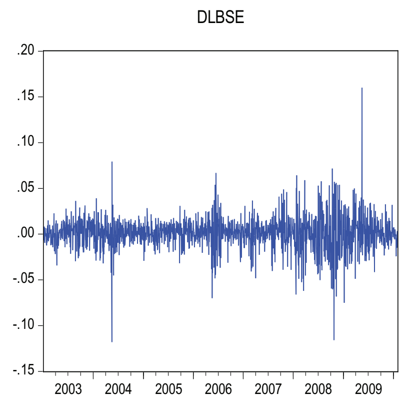
In the study, we have used daily data of two stock price indices, *viz.,* BSE-SENSEX (BSE) and NSE-NIFTY (NSE) covering the period from January 2003 to December 2009. We have estimated 1-day VaR for daily returns of two price indices using univariate GARCH model with proper mean specification and following the FHS approach for VaR estimation. We have also estimated VaR of return using ARMA-GARCH-FHS model and compare the performance of both the VaR estimate. We have used daily S&P500 stock price (SP), daily exchange rate of INR-USD (usd), INR-EURO (euro) and also the gold prices in INR/ounce (gold) for the same period as explanatory variable of the mean equation of the stock prices return. Unit root tests (ADF, PP test) suggest that level series of all the six data series are non-stationary. However, continuous daily return, *i.e.,* log differences of the series (dlbse, dlnse, dlsp, dlusd, dleuro and dlgold) are stationary.

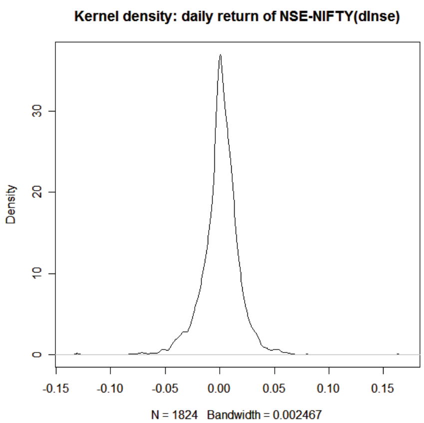
Stylised facts

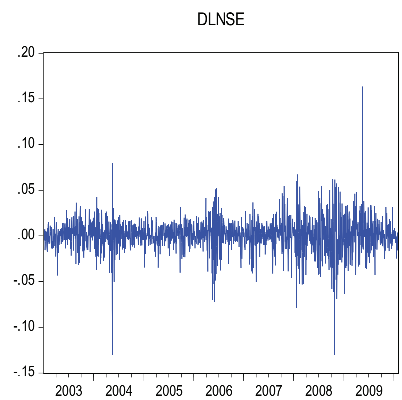
Continuous daily return (log difference) and kernel density of returns on BSE-SENSEX, NSE-NIFTY, S&P500, INR-USD exchange rate, INR-EURO exchange rate and gold prices for the reference period are given in Chart 1 and descriptive statistics are given in table 1. There is a clear presence of fat tails in the return distribution of all the six data series. Various normality test (such as Anderson Darling normality test, Cramer-Von Mises normality test) suggests that the return distributions are not Gaussian normal.

Chart 1: Plot of daily returns and kernel density of Modelling Volatility

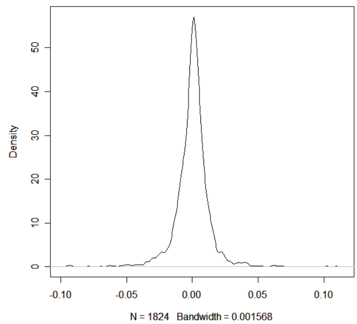




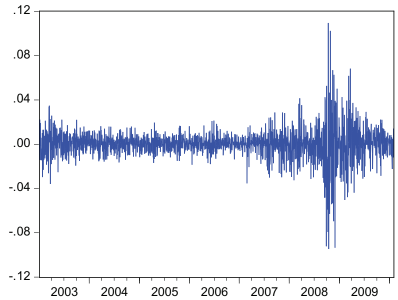




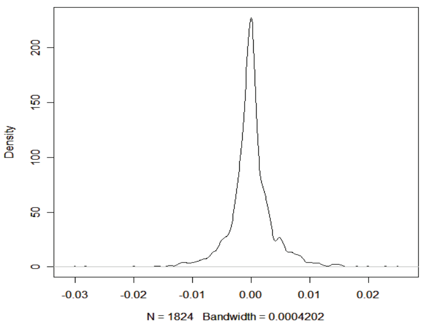
Kernel density: daily return of S&P 500 stockprices(dlsp)



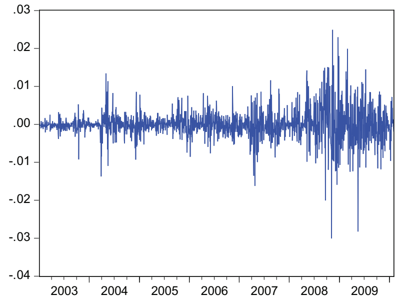
DLSP



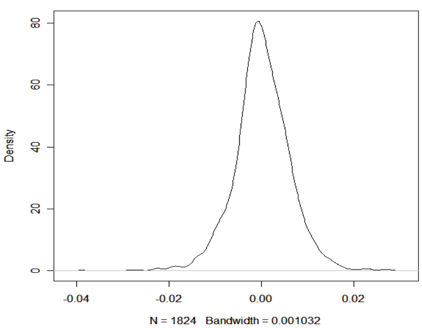
Kernel density: daily return of INR-USD exchange rate(dlUSD)



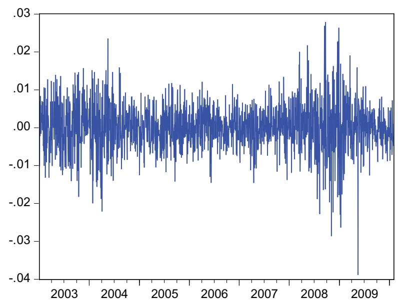
DLUSD



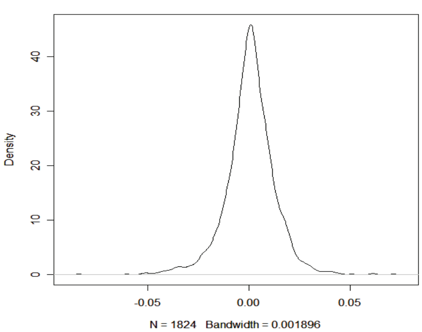
Kernel density: daily return of INR-EURO exchange rate(dleuro)



DLEURO



Kernel density: daily return of GOLD (dlgold)



DLGOLD

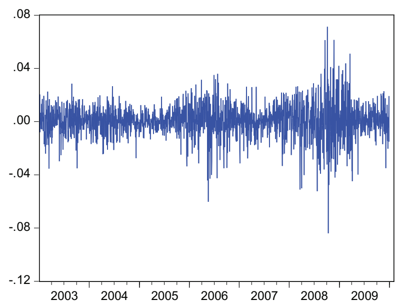


Table 1: Descriptive statistics

	DLNSE	DLBSE	DLSP	DLUSD	DLEURO	DLGOLD
Mean	0.000804	0.000849	0.000105	-2.02E-05	0.000134	0.000617
Median	0.001078	0.001614	0.000799	0	0	0.000865
Maximum	0.163343	0.1599	0.109572	0.024903	0.0279	0.071278
Minimum	-0.13054	-0.11809	-0.0947	-0.03007	-0.03889	-0.08396
Std. Dev.	0.017498	0.017166	0.013291	0.003855	0.006085	0.012485
Skewness	-0.31933	-0.11242	-0.23195	-0.02245	-0.14065	-0.30721
Kurtosis	12.07311	11.0435	15.13967	10.82578	5.58351	6.944578
Jarque-Bera	6370.147	4985.629	11364.19	4715.852	520.0318	1211.227
Sum	1.485895	1.569306	0.193701	-0.03735	0.24755	1.126134
Sum Sq. Dev.	0.56552	0.544288	0.326289	0.027452	0.068397	0.284156
Observations	1848	1848	1848	1848	1848	1824

Equations (2) and (3) present the estimated portfolio model where lag values of (dlbse, dlsp, dlusd, dleuro and dlgold) are used in the mean equation of the GARCH(1,1) model of BSE and NSE, respectively.

Eq (2):

D(LOG(BSE)) = 0.00152 + 0.32558*D(LOG(SP500(-1))) +
(0.00027) *(0.02662)*
0.16716*D(LOG(SP500(-2))) + 0.13393*D(LOG(SP500(-3))) +
(0.026299) *(0.029617)*
0.10005*D(LOG(SP500(-4))) - 0.06044*D(LOG(BSE(-2))) –
(0.027628) *(0.025798)*
0.04891*D(LOG(BSE(-3))) + 0.05722*D(LOG(GOLD(-2))) +
(0.020935) *(0.023705)*
0.15824*D(LOG(EURO(-3))) - 0.24620*D(LOG(USD(-3))) +
(0.053619) *(0.088501)*
0.15767*D(LOG(USD(-4)))
(0.084305)

GARCH = 5.049e-06 + 0.1527939*RESID(-1)^2 + 0.83803454*GARCH(-1)
(5.05E-06) *(0.152794)* *(0.838035)*

Eq (3):

D(LOG(NSE)) = 0.00134 + 0.32468*D(LOG(SP500(-1))) +
(0.000292) (0.027540)
0.16821*D(LOG(SP500(-2))) + 0.10268*D(LOG(SP500(-3))) –
(0.027508) (0.030620)
0.03877*D(LOG(NSE(-2))) + 0.06159*D(LOG(GOLD(-2))) +
(0.026039) (0.025277)
0.17716*D(LOG(EURO(-3))) - 0.34938*D(LOG(DOLLAR(-3))) +
(0.051925) (0.088119)
0.19060*D(LOG(DOLLAR(-4)))
(0.087568)

GARCH = 5.52699e-06 + 0.13072*RESID(-1)^2 + 0.85739*GARCH(-1)
(5.53E-06) (0.130725) (0.857389)

Equation (4) and (5) presents the estimated portfolio model using ARMA-GARCH model of BSE and NSE respectively.

Eq (4):

D(LOG(BSE)) = 0.00161 + [AR(1)=0.52534,AR(2)= -0.87026,
(0.000314) (0.062328) (0.061490)
MA(2)=0.79823,MA(3)=0.12583,MA(1)=-0.42263]
(0.066763) (0.026230) (0.066514)

GARCH = 6.28919e-06 + 0.15594*RESID(-1)^2 + 0.83073*GARCH(-1)
(6.29E-06) (0.155941) (0.830730)

Eq (5):

D(LOG(NSE)) = 0.00160+ [AR(2)=-0.45572,AR(4)=-0.6135
(0.000317) (0.084090) (0.096555)
AR(1)=0.49844,MA(2)=0.42253,MA(4)=0.67288,MA(1)=-0.43386]
(0.134215) (0.079148) (0.092978) (0.127459)

GARCH = 7.61169e-06 + 0.13774*RESID(-1)^2 + 0.84330*GARCH(-1)
(1.12E-06) (0.011880) (0.012652)

Note: **Values given in brackets are the standard error.*

Value at Risk: Results

We have estimated 5 percent 1-day-VaR for both BSE-SENSEX and NSE-NIFTY daily return using univariate GARCH model with proper mean specification as estimated in section 4.2 and following the FHS approach for VaR estimation (Model A). We have also estimated 5 percent VaR for both BSE-SENSEX and NSE-NIFTY daily return using ARMA-GARCH-FHS model (Model B). To estimate the model parameter we have used the daily data from 2nd January 2003 to 30th October 2009 and forecasted dynamically 1-day VaR for the period 2nd November 2009 to 24th December 2009, *i.e.,* for 39 days. Actual returns and forecasted VaR based on both Model A and Model B for BSE-SENSE and NSE-NIFTY are given in Chart 2 and Chart 3, respectively. Out of 39 forecasts of VaR for BSE and NSE, only in one occasion, actual return was less than the VaR estimate (failure rate 1/39) for both model A and model B. However, dispersion of VaR from actual returns is not the same. Let the dispersion of VaR at 5 percent significant level based on model A (${}^{A}VaR_t^{.05}$) from the actual return (r_t) be $D^A=\Sigma(r_t-{}^{A}VaR_t^{.05})^2$ and $D^B=\Sigma(r_t-{}^{B}VaR_t^{.05})^2$ for mode B. It is observed that for both BSE-SENSEX and NSE-NIFTY price indices dispersions are less for model A than model B (D^A_{BSE} =0.02483, D^B_{BSE} =0.03022), (D^A_{NSE} = 0.02603, D^B_{NSE} =0.03049).

Chart 2: Daily return on BSE and corresponding VaR based on Model A and Model B

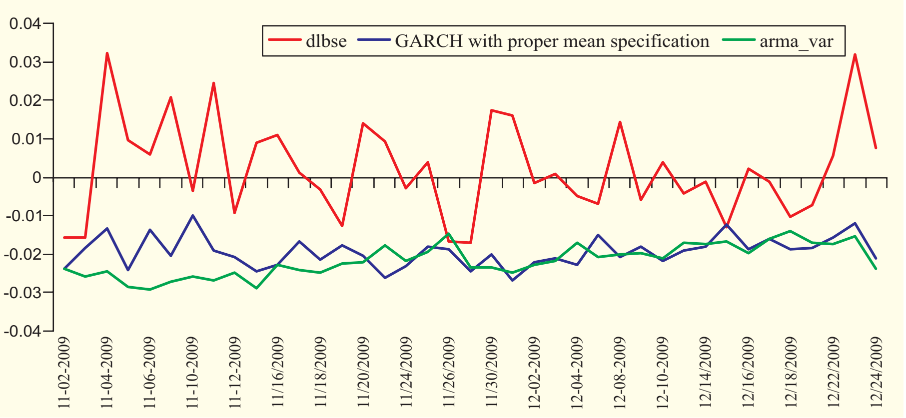
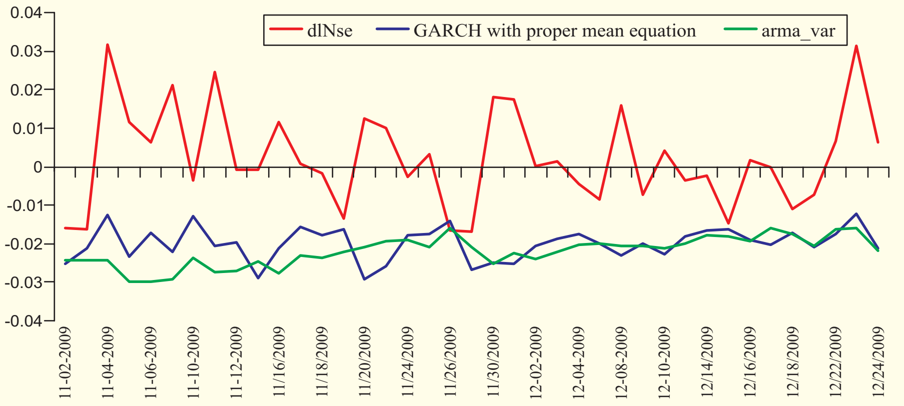


Chart 3: Daily return on NSE and corresponding VaR based on Model A and Model B



RLF and FLF based test as outlined by Sharma, Thomas and Shah (2002) suggests that at 1 percet level of significance, Model A performs better than model B for both BSE-SENSEX and NSE-NIFTY. Diebold-Mariano (1995) test, as outlined in section IV, to test whether losses are statistically significantly different, also indicates that performance of model A is significantly (10 per cent level of significance) better than model B for both the indices BSE-SENSEX and NSE-NIFTY.

Section VI
Conclusion

The paper estimates 1-day VaR taking into consideration the financial integration of Indian capital market (BSE-SENSEX and NSE-NIFTY) with other global indicators and its own volatility using daily return covering the period January 2003 to December 2009. The paper specifies a GARCH framework to model the phenomena of volatility clustering on returns and examines the usefulness of considering lag values of return on (S&P 500, INR-EURO and INR-USD exchange rate, gold price) as proxies to global financial condition in the specification of the mean equation. The paper estimates the VaR of return in the Indian capital market based on two composite methods, *i.e.,* (a) using univariate GARCH model wherein the mean equation uses lag values of return on (S&P 500, INR-EURO & INR-USD exchange rate, gold price) and following the FHS approach (b) using ARMA (for mean equation)-GARCH (to model volatility)-FHS(to estimate VaR) and compare the performance of both the VaR estimates. It is observed that global financial situation (lag values of return on S&P 500, INR-EURO and INR-USD exchange rate, gold price used as proxies to global financial condition) has significant impact on Indian capital market and VaR (as estimated in FHS framework) of return in the Indian capital market based on GARCH method with suitable mean specification outperforms the ARMA-GARCH model of daily return of Indian capital market.

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